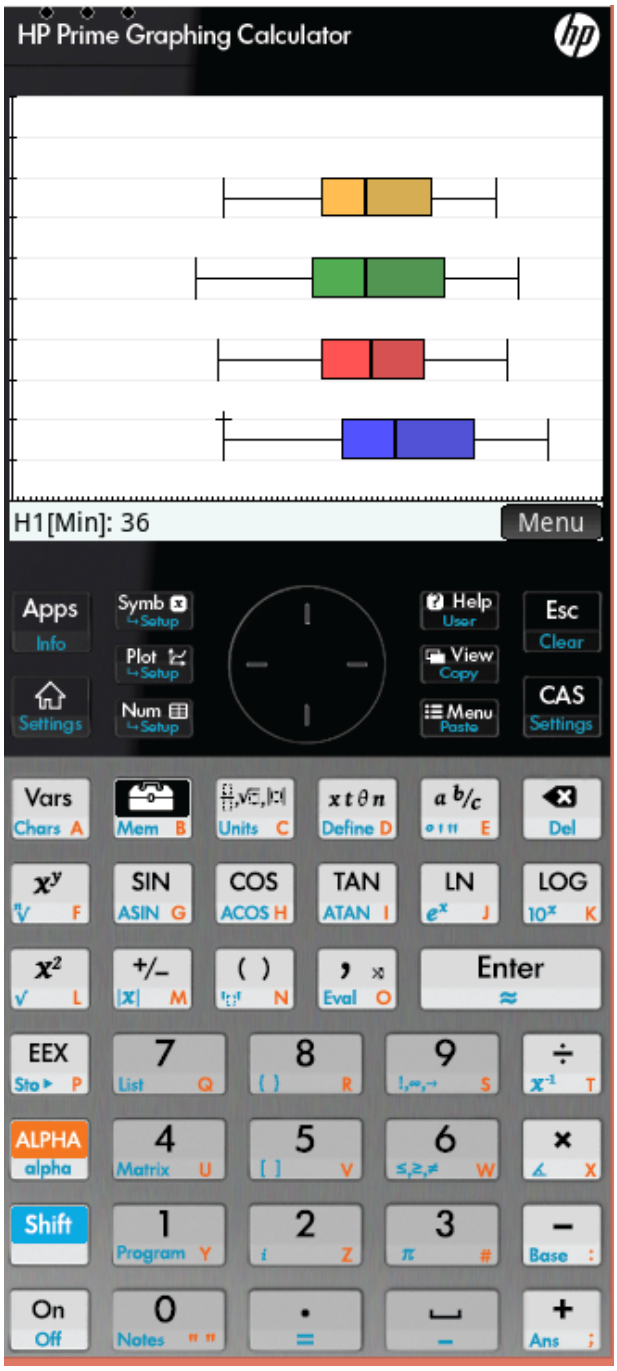
General Mathematics: Units 1&2
HP PRIME activities

Using technology to support mathematics learning

Ian Sheppard
Chris Longhurst



General Mathematics: Units 1&2 - Prime activities
Using technology to support mathematics learning

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This publication makes reference to the Hewlett Packard Prime calculator.

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Introduction

This book comprises a series of activities which are designed to facilitate learning about the mathematics of the Australian Curriculum General Mathematics course and the capabilities of the HP Prime graphing calculator. A key feature of the Prime is the Computer Algebra System (CAS).

Unlike a textbook, the activities cover neither the whole course, nor are they restricted to purely course material. Activities beyond the course content can still assist with solving problems within the course while also increasing student's ability to explore broader mathematical questions, including further mathematics study. In contrast to electronic device manuals this book is about mathematics with detailed instructions on how the technology can be used.

The activities vary in the time needed to complete them. Some are primarily concerned with how to perform a particular technique within a Prime application, and some use the Prime output as the starting point. In others, the Prime is only a small part of the activity.

The activities are arranged into chapters matching the Australian Curriculum topics. Within each topic the activities reflect a possible sequence of learning related to that topic. Many activities can be used as a precursor to formal teaching of the concept thus encouraging a sense-making approach.

Each activity has an aim, linking to curriculum documents, the activity itself and usually a section of *Learning notes*. Fully worked solutions are provided at the end of the text. The learning notes are intended to help with the understanding of concepts, provide more detail or help with instructions for Prime use, provide additional explanations or point to interesting further explorations. As the course progresses more assumptions are made about the skills you have developed and so the instructions become briefer. Where more detailed instructions are required on Prime use, it will often be in the *Learning notes* rather than in the text of the investigation.

The Computer Algebra System (CAS) is very powerful. When we are doing algebraic manipulation with pen and paper, we often decide what to do next by considering the current line of working. Using CAS, however, requires the articulation of steps in words and these words are then the commands for CAS to perform the next step. *Solve*, *simplify*, *factor* and *expand* are examples of these words. Generally, the result is useful, but sometimes you might be unable to find a suitable command. In this case, you may need to work with part of an expression, or even return to pen and paper.

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Knowing when Prime use is quicker or more efficient becomes easier the more experience you have. Working through the activities will help you learn this.

CAS enables us to do is to focus more on what we want to do rather than how do we do it. For example, in a modelling situation we may come across awkward functions that we may not have the tools to deal with by traditional methods. Often, however, CAS will provide an answer so you then evaluate the result, make sense of the result in the real situation and thus demonstrate your understanding of the process of doing mathematics.

I have mainly used the activities in class and as introductions to topics. During these periods there was much discussion between students as they learned the mathematics and enhanced their skills with CAS.

Ian Sheppard

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Chapter 1 Preliminaries

Activity	Prime apps	Key concepts
Basic calculations quiz	Home	Calculate using Prime

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Activity 1 Basic calculations quiz

Aim: Calculate using Prime.

Setting up: refer to Learning notes.

Round decimal answers to 3 decimal places.

Total your answers to Questions 1 – 10.
If all your answers are correct then your total will match the given value.

	Question	Answer
1	16.2×3.1	
2	$7 + 2 \times 3^2 - 15$	
3	$\sqrt{17}$	
4	$\sqrt{9+16}$	
5	$963.1 \div 171.6 - 6.3$	
6	$\sqrt{9} + \sqrt{16}$	
For Q's 7 to 10 use $A = 3.91$, $B = 1.0765$, $C = 6.3$ and $D = 8.9$		
7	Calculate A^2 when $A = 3.91$	
8	Calculate $A^2 - C$	
9	Calculate $\frac{A}{D}$	
10	Calculate $\frac{A+B}{B}$	
	Total Q's 1 - 10	105.003

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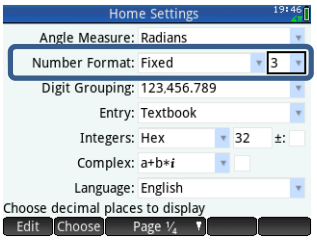
Learning Notes

This activity supports you to perform simple calculations on the Prime, i.e. become familiar with some of the buttons. There are usually a number of ways of performing an operation on the Prime.

Setting to Fixed mode will automatically round answers. However display of questions will also be displayed as decimals with the specified number of decimal places. You may prefer to remain in Standard mode and just round yourself.

Display 3 decimal places

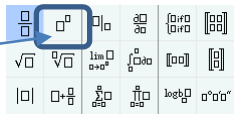
- Press Settings for home mode
- Press Shift Settings to set up home mode as shown. Select Fixed from Number format pull-down menu
- Reset to Standard after completing the activity



To calculate with:

Powers

- Press x^y or
- Press Units and use the template

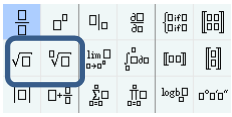


Exit template

- Press the right arrow to exit the template

Square roots

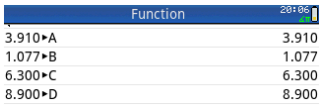
- Press Shift x^2
  or
- Press Units and use the template



Store the values of *A*, *B*, *C* and *D*.

- Tap Settings
- Enter 3.91, tap Sto▸ and press ALPHA Vars Enter
- Enter 1.0765, tap Sto▸ and press ALPHA Mem Enter
- Repeat for all other variables.

You may prefer to just enter the values without storing.



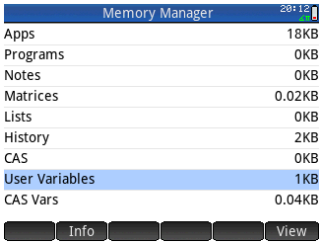
Sum answers

- Scroll to first answer and tap to highlight
- Tap Copy then press Ans +
- Highlight next answer, tap Copy then press Ans +
- Repeat for remaining answers

Clear variables

a good idea when you have finished with them

- Press Shift Mem to access Memory Manager
- Tap on User variables and tap VIEW, Reset and OK



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Chapter 2 Consumer arithmetic

Activity	Prime apps	Key concepts
Percentage quiz	Main	Calculations with percentages
Best buys	Main	Compare prices and values
Maddy's boots	Spreadsheet	Convert between currencies using exchange rates
Maxine's car	Spreadsheet	Investigate the potential running costs of owning a car

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Activity 2 Percentage quiz

Aim: Solve percentage problems.

- Round off decimal answers to 2 decimal places.
- Total your answers to Questions 1 – 10.
- If all your answers are correct then your total will match the given value.

<table>
<tr><th></th><th>Question</th><th>Answer</th></tr>
<tr><td>1</td><td>Determine the commission of 0.23% on $12 340</td><td></td></tr>
<tr><td>2</td><td>An apartment is bought for $463 500 and sold for $521 000. What is the percentage increase?</td><td></td></tr>
<tr><td>3</td><td>Wages are to increase by 3.2%. What will a wage of $17.34 per hour become?</td><td></td></tr>
<tr><td>4</td><td>Wages have increased by 3.2%. How much did a person currently getting $17.34 per hour receive before the increase?</td><td></td></tr>
<tr><td>5</td><td>A jacket is for sale for $120. Adam buys it for 25 % off and then sells it to George for a 25 % profit. What did George pay?</td><td></td></tr>
<tr><td>6</td><td>64.3 increased by 5.2% is</td><td></td></tr>
<tr><td>7</td><td>$796.90 decreased by 31% is</td><td></td></tr>
<tr><td>8</td><td>What is the tax on an item that sells for $550 including GST of 10%?</td><td></td></tr>
<tr><td></td><td colspan="2">Questions 9 and 10 relate to Ann, a financial advisor.
<table>
<tr><th>Portfolio value</th><th>Fee</th></tr>
<tr><td>0 - $200 000</td><td>$3000</td></tr>
<tr><td>$200 001 to $500 000</td><td>commission of 1.5%</td></tr>
<tr><td>More than $500 000</td><td>$3500 plus a commission of 0.8%</td></tr>
</table></td></tr>
<tr><td>9</td><td>What will Ann charge for a portfolio of $450 000.</td><td></td></tr>
<tr><td>10</td><td>What will Ann charge for a portfolio of $650 000</td><td></td></tr>
<tr><td></td><td>Total Q's 1 - 10</td><td>16305.48</td></tr>
</table>

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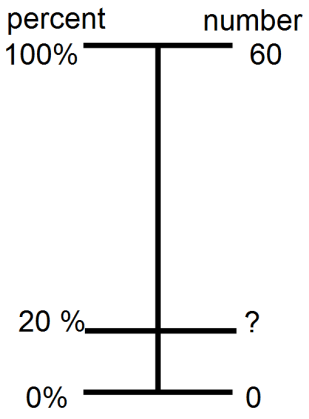
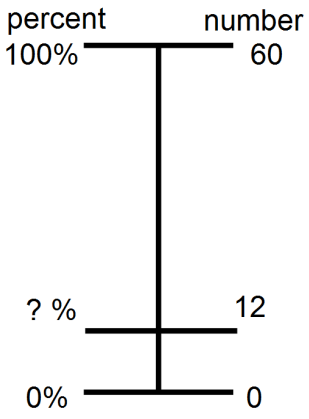
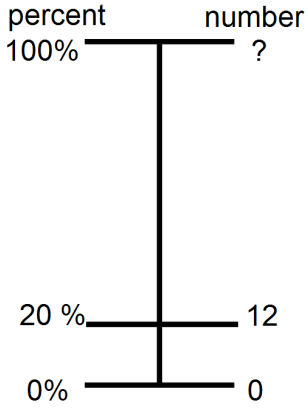
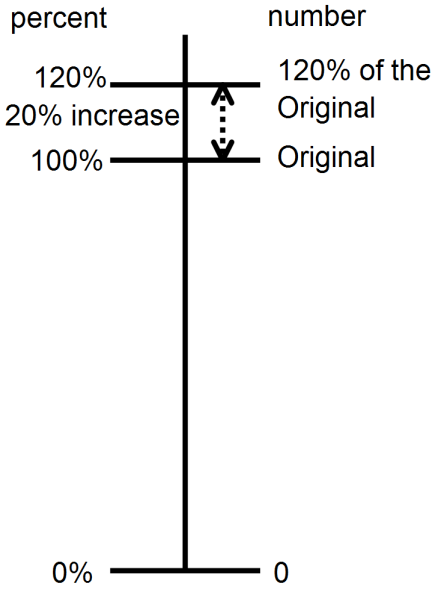
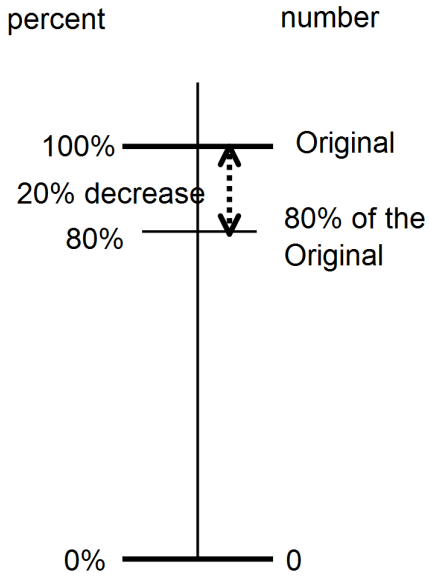
Learning Notes

This is a visual summary of how to work with percentages:
12 out of 60 books on a shelf are sports books, i.e. 20% of the books are sport books.

Finding the part	Finding the percent	Finding the whole
		
What is 20% of 60 books? $60\times0.2=12$ books	12 out of 60 is what percentage? $\frac{12}{60}\times100=20\%$	12 is 20% of how many? $\frac{12}{0.2}=60$ OR 1% is $\frac{12}{20}=0.6$ so 100% is $0.6\times100=60$.

Percentage Increase	Percentage Decrease
	
The new amount is 120% (100 + the increase) percent of the original.	The new amount is 80% (100 – the decrease) of the original

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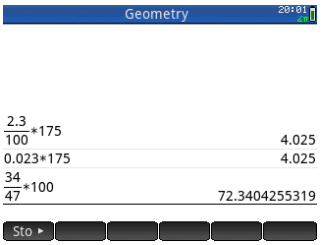
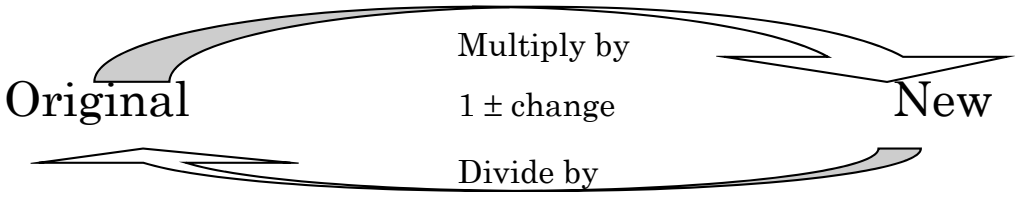
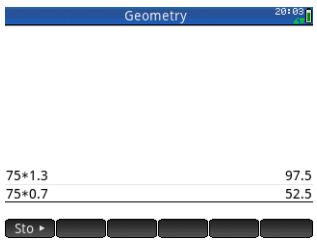
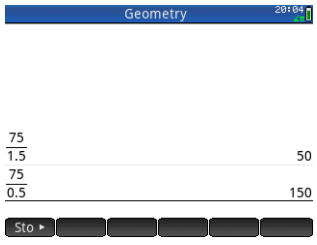
Percentage means **per hundred** or how many out of every 100. It is important to decide what the percentage is out of. This is the whole or 100%

% of an amount	**Example 1** *How much is 2.3% of 175?* It is 2.3 out of every 100 $\frac{2.3}{100}\times 175$ or $0.023\times 175 = 4.025$	
Amount - %	**Example 2** *Jane gets 34 out of a possible 47 for her Maths test. What is her percentage?* 34 out of 47 is $\frac{34}{47}$ As a percentage: $\frac{34}{47}\times 100\% = 72.3\%$	

Percentage change problems : Original amount is 100%

New amount is 100% + increase OR 100% – decrease



Original to New **Original** Multiply by (1 ± change)	**Example 3** *Increase 75 by 30%* New amount is 130% of original i.e. 75×1.3 **Example 4** *Decrease 75 by 30%* New amount is 70% of original i.e. 75×0.7	
New to Original **New** divided by (1 ± change)	**Example 5** *What increased by 50% is 75?* New amount is 150% of the original Original = 75 ÷ 1.5 **Example 6** *What decreased by 50% is 75?* New amount is 50% of Original Original = 75 ÷ 0.5	

Do calculations in the Home screen. Press Settings

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Activity 3 Best buys

Aim: Compare prices and determine the value for money.

Fill in the blanks in each statement and hence determine the better (or best) choice.

1. Fantasy Footy Tipping

Hi Jesse, Outstanding tipping for Round 5. You scored 5 points from the 7 games.	Hi Jesse, Great tipping for Round 8. You scored 6 points from the 9 games.

a) In Round 5 Jesse scored _____ points per game.

b) In Round 8 Jesse scored _____ points per game.

c) In which round was Jesse's tipping more accurate? _____

2. Land For Sale

Shell Cove subdivision
Three superior blocks

Block A	**Block B**	**Block C**
334 m^2	361 m^2	352 m^2
$224 000	$245 000	$240 000

a) Block A costs $ _____ per square metre.

b) Block B costs $ _____ per square metre.

c) Block C costs $ _____ per square metre.

d) Which block is the best buy if the buyer wants land for the least cost per square metre?

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3. Debbie is responsible for calculating the unit pricing to be displayed in the supermarket. Complete the table for Debbie.

Item	Quantity	Price	Unit Price Quantity	Price per unit
Cooking Oil	750 mL	$6.43	1 L	
Fruit Juice	2.5 L	$7.96	1 L	
Chocolate bar	65 g	$1.50	100 g	
Flavoured Milk	600 mL	$2.50	1 L	
Bread	650 g	$4.45	1 kg	
Peanut Butter	450 g	$5.20	100 g	
Flour	2 kg		1 kg	$3.45

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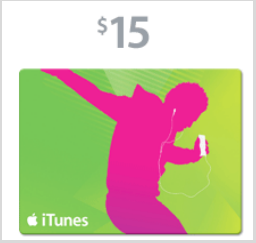
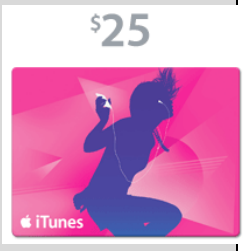
Learning Notes

Do calculations in the Home screen. Press Settings

Is 12 songs for $15 a better buy than 18 songs for $25?



12 songs



18 songs

Jesse thinks:

12 songs for $15
Means each song costs $15 / 12 = $1.25
18 songs for $25
Means each song costs 25 / 18 = $1.39
$1.25 per song is less than $1.39 per song.

Debbie thinks:

12 songs for $15
Means 12 / 15 = 0.8 songs per $1
18 songs for $25
Means 18 / 25 = 0.72 songs per $1
0.8 songs per $1 is better than 0.72 songs per $1.
Yes 12 songs for $15 is a better buy.

Both Jesse and Debbie come to the same conclusion:
12 songs for $15 is a better buy than 18 songs for $25.

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Activity 4 Maddy's boots

Aim: Develop and use a currency converter spreadsheet.

Maddy is shopping online. She sees these boots that come from the United States.

How much will it cost to buy them?

She needs to "buy" US $16.58 and then there will be postage or shipping charges as well.

Exchange rate table Units of foreign currency per A$ on 20 Dec 2013	
United States dollar	0.8871
Japanese yen	92.67
European euro	0.6506
South Korean won	942.06
Singapore dollar	1.1236
UK pound sterling	0.5422
New Zealand dollar	1.0808
Thai baht	28.88
Malaysian ringgit	2.9128
Indian rupee	55.25
Indonesian rupiah	10867
Vietnamese dong	18722
South African rand	9.2170

Source: http://www.rba.gov.au/statistics/frequency/exchange-rates.html

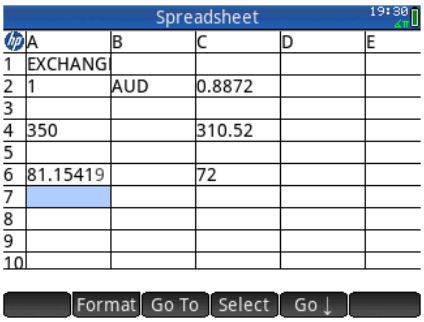
The exchange rate table shows how much of the foreign currency can be bought with A$1.00.

This activity requires you to build a spreadsheet to convert between currencies.

1. Build the spreadsheet shown

   Step by step instructions are in the Learning notes

   Ensure the correct formulas are used in cells C4 and A6.



   a) Describe what happens when the number in cell A4 is changed.

   b) Describe what happens when the number in cell C6 is changed.

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2. Complete the table using the rates shown above

A$ (Australian dollars)	Other currency
A$214	Yen
	75 Euros
A$23.50	won
A$1093	Pound sterling
	200 000 Vietnamese dong
A$75	rupiah

3. Maddy wants to buy a new jacket online. It is advertised at $45.70 US. How much will it cost in AUD?

4. Repeat Q2 using current rates.
E.g. use http://www.rba.gov.au/statistics/frequency/exchange-rates.html

A$ (Australian dollars)	Other currency
A$214	Yen
	75 Euros
A$23.50	won
A$1093	Pound sterling
	200 000 Vietnamese dong
A$75	rupiah

5. Challenge: convert $45.20 US into Malaysian ringgit.

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Learning notes:

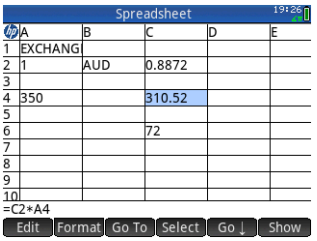
Detailed instructions to build the spreadsheet

Open spreadsheet app

- Press Apps Info
- Choose Spreadsheet

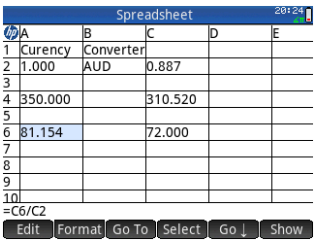
Enter values

- Tap cell A1
- Press ALPHA alpha 0 Notes " " Enter a heading *Exchange rate*
- Press Enter ≈
- Tap in cell A2 and enter *1* Enter ≈
- Tap in cell B2 and press ALPHA alpha 0 Notes " " then *AUD* Enter ≈
- Tap in cell C2 and enter *0.8872*
- Tap in cell A4 and enter *350* (or any number)
- Tap in cell C6 and enter *72* (or any other number)



Enter formulas

- Tap in cell C4 and enter the formula =C2*A4 ÷ = ALPHA alpha Units c 2 i Z × ∠ x ALPHA alpha Vars Chars A 4 Matrix U Enter ≈
- Tap in cell A6 and enter =C6/C2
- The formula is displayed at the bottom of the screen.



Q5 Convert $45.20 US to $AUD first.

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Activity 5 Maxine's car

Aim: Build spreadsheets as a financial model.

Maxine wants to buy a car and wants to make sure she can meet the costs of running and maintaining the car. She can afford up to $50/week.

Her research is summarised in the table below.

Item	Assumptions
Usage	220 km per week
Fuel (petrol)	Fuel is currently $1.65 / litre. The car uses 7 litres every 100 kilometres.
Tyres	A new set costs around $390 and lasts 50 000 km
Service	$150 every 5000 km
Repairs	$500 per year
Registration	$650 per year
Insurance	For third party property it is $235 per year

1. How much will Maxine pay for petrol each week?

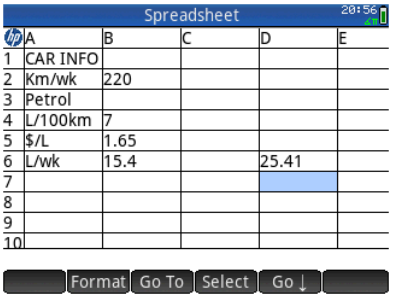
   a) How many litres are required for 220 km?

   b) How much does that quantity of petrol cost each week?

2. Build a spreadsheet to do this calculation.

   a) What is the formula in cell B6?

   b) Compare your answers to Q1 a) and Q2 a)



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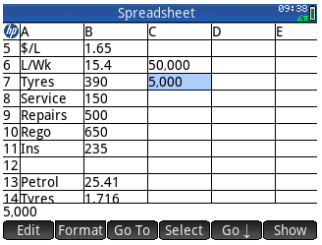
3. If Maxine chooses a more fuel efficient car that only uses 4.6 L per 100 km, what will petrol cost per week?
(Hint: Tap in cell B4 and change the number. Change it back before answering Q4.)

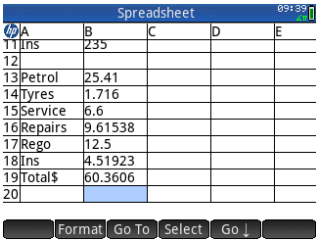
4. Modify the spreadsheet as shown.

- (Use the information shown in the table on the previous page.)
- Cells B13 to B19 each require a formula.

a) Use the spreadsheet to determine Maxine's total running costs per week.

b) Hence determine whether or not she can afford the car.





5. Research a car of your own choosing, adjust the values in your spreadsheet and determine the running cost per week.

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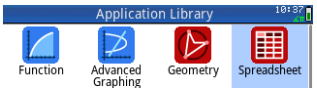
Learning notes

Detailed instructions for complete spreadsheet

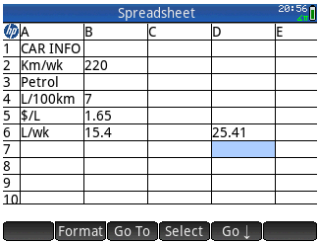
Open spreadsheet

- Press Apps Info tap the Spreadsheet button



Enter values

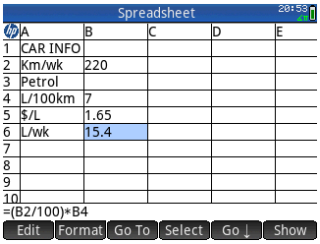
- Tap cell A1
- Press ALPHA alpha 0 Notes " " Enter a heading *CAR INFO* Enter ≈
- Tap in cell A2 Press ALPHA alpha 0 Notes " " and enter *Km/wk*
- Tap in cell A3 ALPHA alpha 0 Notes " " and enter *Petrol*
- Tap in cell A4 ALPHA alpha 0 Notes " " and enter *L/100km*
- Tap in cell A5 ALPHA alpha 0 Notes " " and enter *$/L*
- Tap in cell A6 ALPHA alpha 0 Notes " " and enter *L/wk*
- Tap in cell B2 and enter 220
- Tap in cell B4 and enter 7
- Tap in cell B5 and enter 1.65



Enter formulas

To enter a formula you must start with an =

- Tap in cell B6 and enter the formula = B2/100*B4
  Press =, enter B2/100*B4 and press Enter ≈ (divide the 220 by 100 to calculate how many hundreds of kilometres and then multiply by the number of litres used per 100 km.)
- Tap in cell D6 and enter the formula =B6*B5



To convert cost per year to cost per week divide by 52, e.g. for repairs: =500/52.

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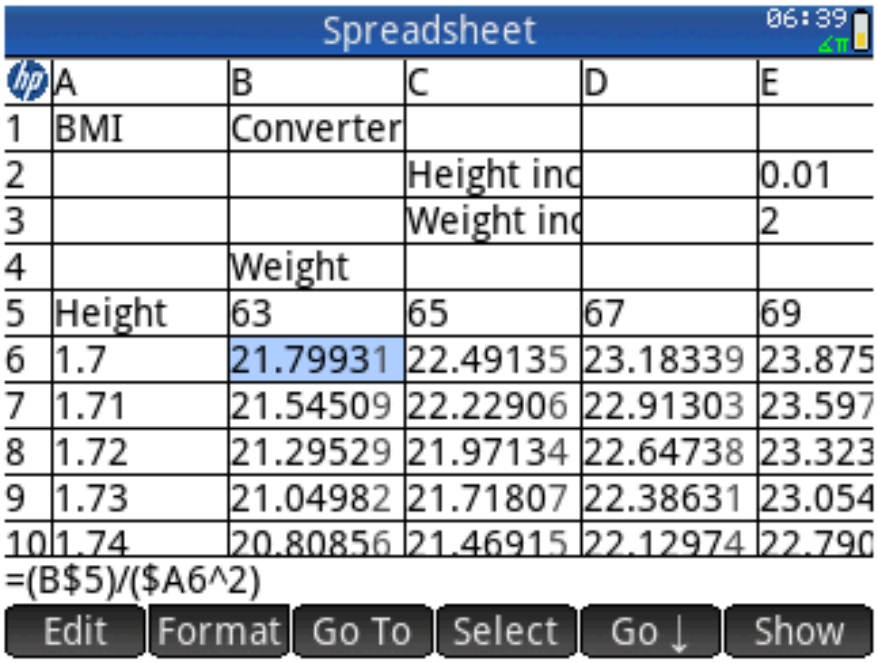
Chapter 3 Algebra and matrices

Activity	Prime apps	Key concepts
Algebra quiz	Main	Store values and substitute in algebraic expressions
Currency trade	Solve in Main	Rearrange formulae
Taxi charges	Main	Substitute in formulae and solve problems
Body mass index	Spreadsheet	Generate two by two tables for formulae
Matrix arithmetic	Main	Operate with matrices on Prime
Premiership table	Main	Calculations with matrices



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Activity 6 Algebra quiz

Aim: Substitute into algebraic expressions.

Setting up: refer to Learning Notes

Store these values in your Prime: $A = 3.91$, $B = 1.0765$, $C = 6.3$ and $D = 8.9$.
Round answers to 3 decimal places.
Total your answers to Questions 1 – 10 and compare to the given value.

		Hint	Answer
1	Evaluate A^2	ALPHA Vars x^2	
2	Evaluate $A^2 - C$	(–) ALPHA	
3	Evaluate $A \div D$		
4	Evaluate $(A + B) \div B$		
5	Find E if $E = 10 - (A + B)$	Enter expression $10 - (A + B) \blacktriangleright E$	
6	Find F if $F = \dfrac{\sqrt{A}}{B}$		
7	Find G if $G = \dfrac{E}{F}$		
8	Find H if $A = H - B$	solve(A=H–B, H)	
9	Find K if $E = A + K(B + C)$		
10	Find L if $G + K = L - B$		
	Total Q's 1 - 10		48.019

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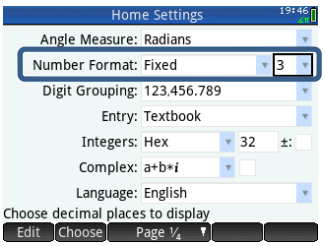
Learning notes

Ensure decimal answers will be displayed

- Press Shift Settings
- Shift et Number Format to Fixed and 3 decimal places.
  (Toggle to Standard for exact answers)



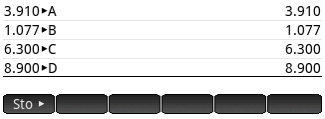
Set up

- Press Settings to select the Home screen
- Press Shift Esc Clear to clear the screen if desired

Store variables

- Enter the number e.g. 3.91
- Tap Sto ▸ ALPHA Vars to enter the variable as A
- Press Enter

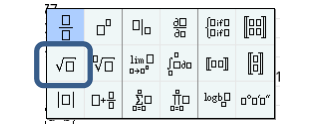
Repeat for the other variables



Hints for calculations

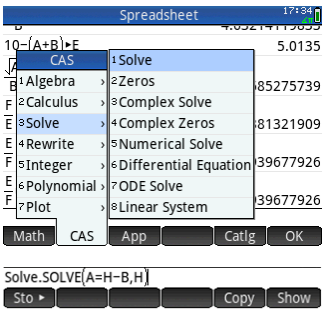
Store each variable as it is calculated e.g. for Q5:
1 0 − ( ) ALPHA Vars − ALPHA Mem press right arrow, tap Sto ▸ and press ALPHA x^2

- Press Units to access templates for square root



- To solve for a variable e.g. Q8
  Press Mem to access CAS commands
  Tap CAS > Solve > Solve

Complete entry of equation:
ALPHA Vars Shift = ALPHA COS − ALPHA Mem , Eval ALPHA COS Enter



Q8 The solve command will not store the value so to use in later calculations store the answer, e.g. ans►K

While it is not necessary to save the variables it is likely to lead to fewer errors and is a useful approach to develop.
Using capital letters on Prime is like using a single letter when writing algebra on paper.

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Activity 7 Currency trade

Aim: Rearrange formulae to use in spreadsheet.

Part A Currency trading

In Activity 3 you created a spreadsheet to convert money between $A and other currencies. This could be written as a formula $f = e \times a$ where f is the value of the foreign currency, a is the amount in Australian dollars and e is the exchange rate.

1. Convert A$300 to US$ where the exchange rate is 0.90 US per A$1.

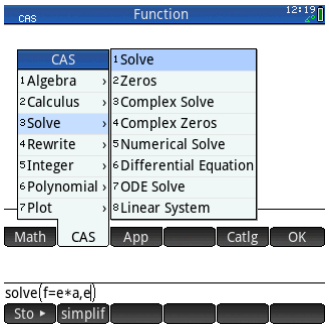
2. What is the exchange rate (the number of ringgits equivalent to A$1) if A$213 buys 598 ringgit?

For Q2 you probably divided 598 by 213.

3. Rearrange the formula to make e the subject.

Rearrange the formula

- Press CAS Settings to open the CAS screen
- Tap Mem
- Select Solve from the CAS tab
- Enter the formula
- Press , and then the variable you want to solve for (make the subject of the formula) and press Enter



a) Record your answer.
(Note: This is the formula you would use for a spreadsheet set up to determine the exchange rate.)

b) Make a the subject of the formula and record the answer.

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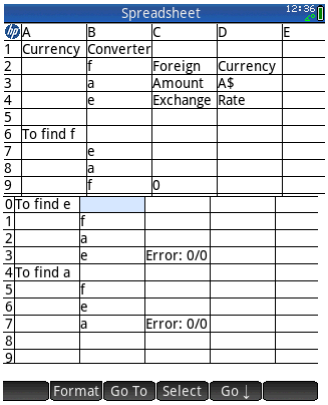
4. Create the spreadsheet shown to calculate the missing quantity knowing any two of f, e and a.

   See Learning notes for more detail.
   In the first part it is intended that numbers be put in cells C7 and C8 and then a formula in C9 calculates the value of the foreign currency.

   In the second part enter values for f and a and then calculate e.

   In the third part enter values for f and e and then calculate a.



State the formulas required in cells

a) C9

b) C14

c) C19

Part B: Measurement formulas

5. Rearrange each formula to determine the:

   a) radius of a circle given its circumference, $C = 2\pi r$

   b) width of a rectangle given its perimeter and length, $P = 2(l + w)$

   c) side of a square given its area, $A = l^2$

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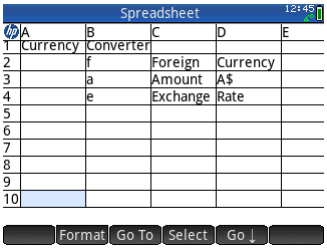
Learning notes

Q3 In the CAS screen we are using lower case letters. If you use capitals Prime will treat these as numbers and you will not be able to do the algebra.

See earlier activities for general spreadsheet use.

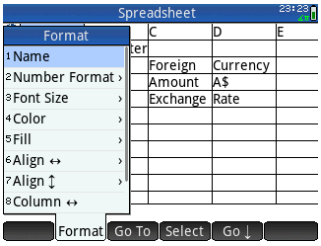
Set up headings/labels

- Tap in A1 [ALPHA] [0] and enter *Currency*
- Tap in A2 [ALPHA] [0] and enter *Converter*
- Tap in B2 [ALPHA] [0] and enter *f*
- Tap in B3 [ALPHA] [0] and enter *a*
- Tap in B4 [ALPHA] [0] and enter *e*
- Tap in C2 [ALPHA] [0] and enter the meaning for symbol *f*, e.g. amount of foreign currency. Note if the text does not fit the cell enter the overload into the next cell
- Similarly for cells C3 and C4 as shown



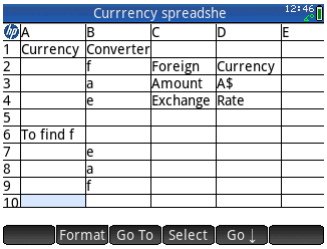
Adjust column widths

- Tap on the top of column e.g. A
- Tap Format, select column ↔
- pinch open or close to adjust width



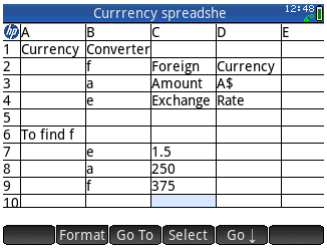
Knowing *e* and *a* calculate *f*

- Tap in A4 and enter *To find f*
- Tap in B5 and enter *e*
- Tap in B6 and enter *a*
- Tap in B7 and enter *f*
- Repeat for the other two sections



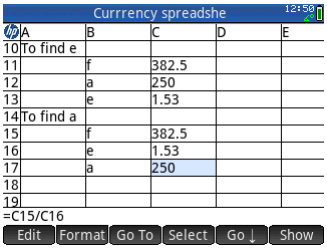
Enter formulae

- Tap in cell C9
- Press [Shift] [÷] C7*C8 [Enter]
- Put in some values for *a* and *e* to check formula is correct



Add sections to calculate other values

- Add the appropriate labels as shown
- Enter formulas in C13 and C17, using your formulae from Q3
- Put in some values to check formula is correct



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Activity 8 Taxi charges

Aim: Substitute in formulae and solve equations.

Omar is a Sydney taxi driver. Charges are just determined by the meter installed in his taxi but he is curious as to the calculations involved. Perhaps if he understands this he can work smarter. He knows that the fare he can charge depends upon

- the distance,
- the time of day and
- how long is spent waiting for a booking or stuck in traffic.

In 2013 the maximum fare that could be charged by Sydney taxis (ignoring any tolls) was given by the formula

$$F = H + R \times D + 0.921M \text{ where}$$

H is the hiring charge or flagfall \$3.50 or \$6.00 at night (10pm to 6am).
R is the per kilometre rate \$2.14 per kilometre or 20% more at night.
D is the distance in km.
M is the number of minutes the taxi is moving at less than 26km/h.

1. What is the maximum charge per km travelled at night?

2. Complete this table

Day/night	hire charge H	per km rate R	kms travelled D	minutes waiting M	maximum fare F
day	\$3.50	\$2.14	8.6	2	
night	\$6.00		12	0	
day			14	5	
night			4.5	1	
day				12	\$41.37
night			7.2		\$40.15

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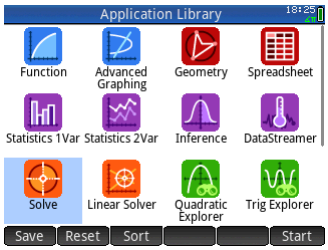
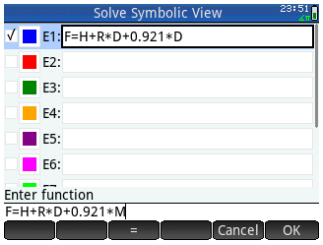
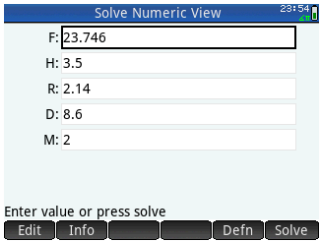
Learning Notes

Open the SOLVE app • Press Apps Info and tap Solve	
To enter the formula • In E1 Enter formula Note: variables are in capital letters •	
Solve for unknown • Press Num Setup and enter the values for variables • Toggle to unknown and tap Solve (there can only be 1 unknown value and it doesn't matter whether or not there is a value thee Prime will recalculate)	

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Activity 9 Body mass index

Aim: Create a two-way table of values.

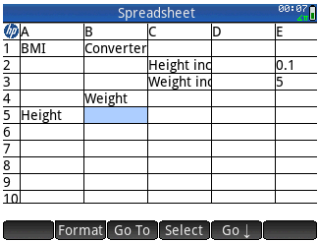
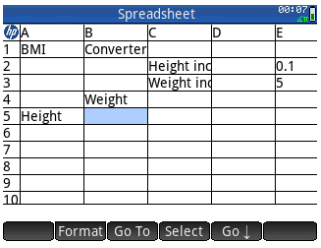
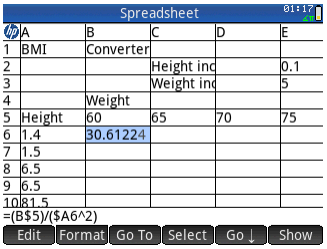
The Body mass index (BMI) is sometimes used as a health indicator.
It is calculated by dividing a person's weight (in kg) by the square of their height in metres.

$$\text{BMI} = \frac{\text{weight}}{\text{height}^2}$$

1. Estimate your own BMI.

2. What is regarded as a healthy BMI for Australians of your gender?
Refer to this or other sources: http://www.heartfoundation.org.au/healthy-eating/Pages/bmi-calculator.aspx

Build a spreadsheet to display a two-way table or chart for looking up BMI values.

Enter headings • Tap in cell A1, press [ALPHA] [0 Notes] and enter *BMI* • Tap in cell B1 and enter *Converter* (remember the quotes for labels) • Enter *height inc* in cell C2 • Enter *weight inc* in cell C3 • Enter *weight* in cell B4 • Enter *height* in cell A5	
Set increments • Tap in cell E2 and enter 0.1 This is the difference in height between successive rows • Tap in cell E3 and enter 5 This will be the difference in weight between successive columns	
Enter height column labels • Tap in cell A6 and enter 1.4, or whatever you want for the smallest height • Tap in cell A7 and enter the formula =A6+E2. • Tap in cell A7 • Press [Shift] [View Copy] • Set the range to A7:A20 by dragging across • Press [Shift] [Menu Paste] select 1 then formula [Enter]	

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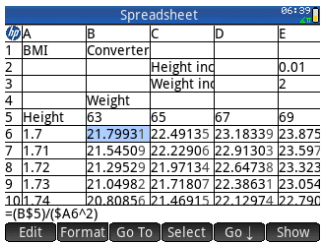
Fill table • Tap in cell B6 and enter the formula =B$5/($A6^2) • Tap in cell B6 • Edit and copy to cells B7 to B20 • Copy to cells C6 to C20 • Continue to column M	

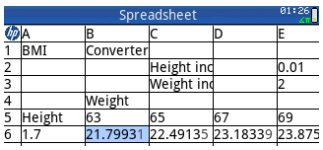
3. Set up your spreadsheet to display values so you are able to fill in the grid.

Height	Weight (kg)			
	63	65	67	69
1.70				
1.71				
1.72				
1.75				

4. Complete the table

Height	Weight	BMI
1.78 m	75 kg	
164 cm	62 kg	
1.66 m		18.5
	85 kg	25

Learning notes

Q2	**Copy formulae** (Column A) • Tap in cell A7, press Shift View Copy to copy • Tap in cell A7 and hold • When Select changes to Sel• drag to select cells A7 to A20 • Tap in cell A7 and press Shift Menu Paste to Paste and select 2Formula	
Q3	**Adjust spreadsheet values** • Set the increment in E2 to 0.01 • Set the increment in E3 to 2 • Change the start height in A6 to 1.7 • Change the start weight in B5 to 63	
Q4	**Adjust values as in Q3**	

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Activity 10 Matrix arithmetic

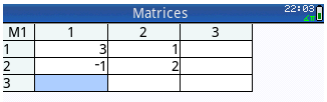
Aim: Perform matrix calculations on PRIME

Enter and store the following matrices:

Enter and store the matrices

$$\mathbf{A}=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\ \mathbf{B}=\begin{bmatrix}1 & 5\\ 2 & 3\end{bmatrix},\ \mathbf{C}=\begin{bmatrix}-1 & 3\\ 2 & -2\\ 5 & 1\end{bmatrix},\ \mathbf{D}=\begin{bmatrix}1 & 2 & -2\\ 4 & 2 & -1\\ 3 & -1 & 2\end{bmatrix}$$

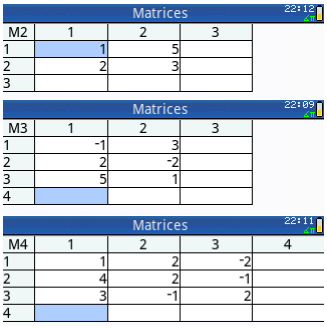
- Press Shift 4 Matrix to open Matrix catalogue
- Tap M1 and complete entries as shown



Enter matrix B as M2 ,

- Press Shift 4 Matrix to open Matrix catalogue
- Tap M2 and complete entries as shown

Enter matrices C as M3 and D as M4



Perform the following calculations (if possible), and record the output.
Where a calculation is not possible, explain why.

1. A + B

2. 3A

3. 2B

4. 2B + 3A

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5. A + C

6. B + 2D

7. A×B

8. B×A

9. BC

10. D×C

11. C×D

12. A^2

13. C – B×A

Learning notes

Matrices can only be added (or subtracted) when they have the same size, i.e. the same number of rows and the same number of columns.
For multiplication the number of columns in the first matrix must be the same as the number of rows in the second matrix. A reason for defining matrix multiplication in this way is demonstrated in the next activity.

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Activity 11 Premiership table

Aim: Calculate with matrices.

Part A

Matt and Siji both follow London clubs in the English Premier League (EPL). This table shows the results of those clubs before 14th December 2013.

Arsenal is on top with 10 wins, 1 draw and 2 losses in their first 13 games.

Position	Team	P	W	D	L	GF	GA	GD	Pts
1	Arsenal	13	10	1	2	27	10	17	31
3	Chelsea	12	7	3	2	21	10	11	24
9	Tottenham Hotspur	12	6	2	4	9	12	-3	20
15	West Ham United	13	3	4	6	12	14	-2	13
18	Fulham	13	3	1	9	11	24	-13	10
20	Crystal Palace	13	2	1	10	7	22	-15	7

1.

a) Complete a matrix for the number of wins, draws and losses for each of the London clubs.

	Won	Drawn	Lost
Chelsea	7		
Tottenham Hotspur		2	
Arsenal			
West Ham United			
Crystal Palace			10
Fulham			

b) How many rows in this matrix?

c) How many columns in the matrix?

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2. Results for the next month are:

$$\begin{pmatrix} 5 & 2 & 3 \\ 7 & 0 & 3 \\ 4 & 5 & 2 \\ 4 & 3 & 2 \\ 5 & 1 & 3 \\ 2 & 2 & 6 \end{pmatrix}$$

Note: the row and column labels are understood to be the same as in Q1.
In this period:

a) how many games has Crystal Palace won?

b) how many games has Tottenham drawn?

c) how many games has Arsenal played?

3.

a) Add the two matrices together.

b) What does this matrix represent?

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Part B

Results before Christmas	Results end of season
$\begin{matrix} & W & D & L \\ \text{Arsenal} & 12 & 1 & 4 \\ \text{Tottenham} & 8 & 3 & 5 \end{matrix} = \text{M1}$	$\text{M2} = \begin{pmatrix} 21 & 7 & 10 \\ 20 & 9 & 9 \end{pmatrix}$

4. How many games did:

   a) Arsenal win between Christmas and the end of the season?

   b) Tottenham lose between Christmas and the end of the season?

5. Enter these matrices into Prime and store them.

   a) Record the result of subtracting C from F on your Prime, i.e. $\text{M2} - \text{M1}$.

   b) What does this matrix represent?

Three points are awarded for a win, 1 point for a draw and 0 points for a loss.

6. How many points does each team have at Christmas?

Enter the points for wins, draws and losses as a 3×1 column matrix $\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$ and store the result as matrix M3.

7. Calculate

   a) $\text{M1} \times \text{M3}$

   b) $\text{M2} \times \text{M3}$

8. What do the answers to Q7 represent?

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9. Write your own descriptions of how to:

   a) Add matrices together;

   b) Subtract one matrix from another;

   c) Multiply two matrices together.

Learning notes

See the previous activity *Matrix arithmetic* for instructions on entering and storing matrices.

The activity is intended to provide a context where the definitions for

- matrix addition
- matrix subtraction and
- matrix multiplication

flow naturally from the context.

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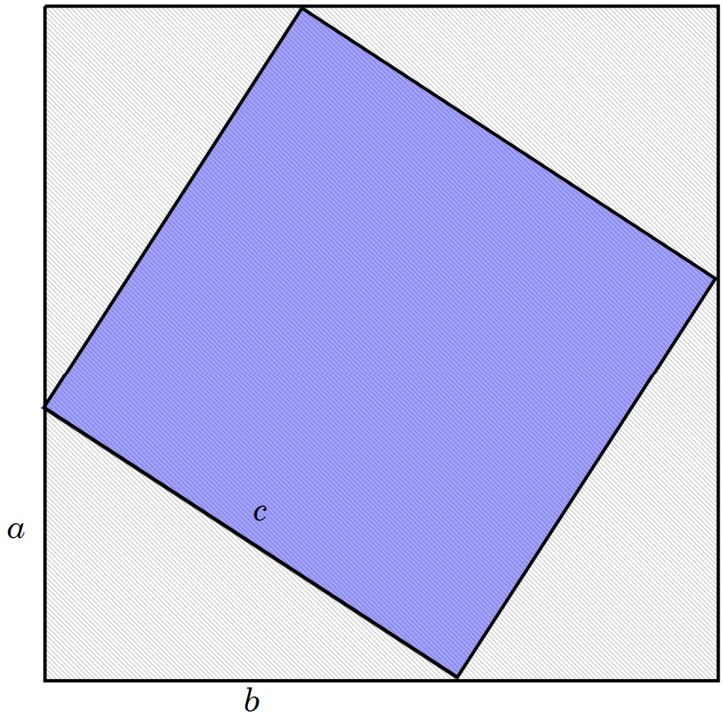
Chapter 4 Shape and measurement

Activity	Prime apps	Key concepts
Pythagoras	Triangle Solver/Solve	Pythagoras' Theorem
Measurement formulas	Spreadsheet	Calculate areas and volumes



Area of outer square = area of inner square + area of four triangles

$$(a+b)^2 = 4\left(\frac{1}{2}ab\right)+c^2$$

$(a+b)^2=4*\frac{1}{2}*a*b+c^2$ $(a+b)^2=2*a*b+c^2$

simplify(Ans) $a^2+2*a*b+b^2=2*a*b+c^2$

Ans−2*a*b $a^2+b^2=c^2$

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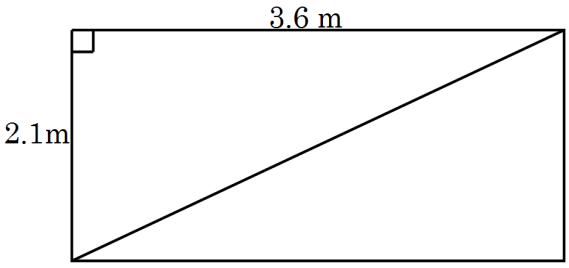
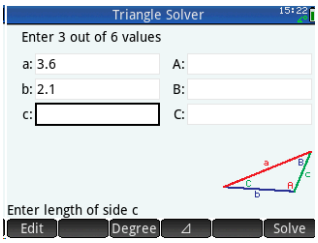
Activity 12 Pythagoras

Aim: Solve Right Triangles using Triangle Solver and SOLVE apps.

1. The farm gate shown below is to have a diagonal brace installed.



Open Triangle Solver app • Press Apps Info • Tap Triangle Solver	
Calculate the hypotenuse • Tap ⊿ for right angled triangle • Note angle C is right angle • Enter values for sides a and b • Tap Solve and read the value of the hypotenuse	

How long should Gerard cut the piece of steel pipe to fit the diagonal?

2. Write solutions to the following problems i.e.
   - Draw a labelled diagram
   - Decide on the Triangle Solver app or Solver app
   - Write your answer, with appropriate units

   a) Luc is flying a kite with a 50 m length of string. The kite is 30 m above the ground. Draw a diagram showing this information. If the string were to break and the kite fall straight to the ground how far would it be from Luc?

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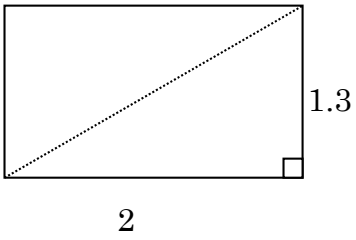
b) What is the distance between the points (–2, 1) and (4, 9)?

c) How much shorter is the direct route from A to B than going around the edge?



d) A concreter checks the slab for squareness. One pair of opposite sides are 20.6 m and the other pair are 30.5 m long. A diagonal is measured to be 37.0 m. Is the corner of the slab a right angle? Explain your answer.

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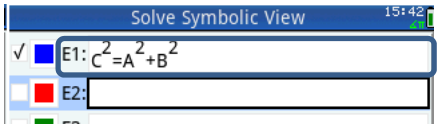
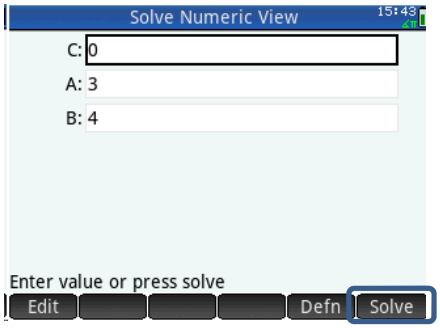
Learning notes:

Using Solve to do Pythagoras

Open Solve app • Press Apps Info • Tap Solve	
Enter the formula • In E1 press ALPHA alpha, Units, x^2, Shift, ÷, ALPHA alpha, Vars Chars A, x^2, + Ans, ALPHA alpha, Mem B, x^2 to enter the formula	
Solve • Press Num Setup and enter the known values • Tap Solve	

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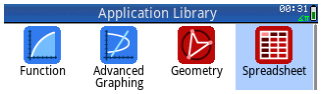
Activity 13 Measurement formulas

Aim: Use a spreadsheet to work with measurement formulas.

Build a spreadsheet to calculate surface area and volume of a cylinder.

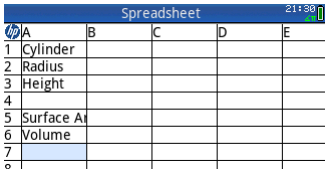
Open spreadsheet

- Press Apps tap the Spreadsheet button



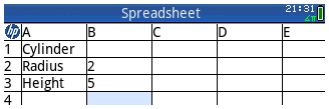
Enter labels

- Tap cell A1
- Press ALPHA 0 Enter a heading *Cylinder* Enter
- Enter labels in cells A2, A3, A5 and A6



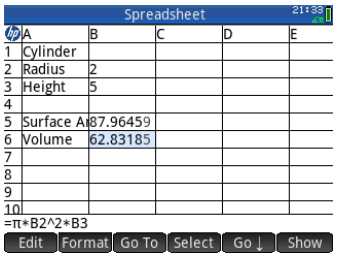
Enter values for height and radius

- Tap in cell B2 and enter the radius of the cylinder
- Tap in cell B3 and enter the height of the cylinder



Enter formulas

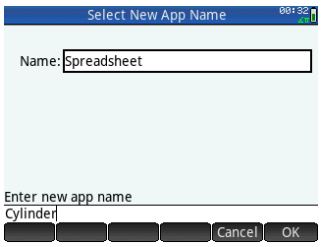
- Tap in cell B5 and enter formula $=2\times\pi\times B2\times(B2+B3)$
- Tap in cell B6 and enter formula $=\pi\times B2\text{^}2\times B3$



Save your spreadsheet

- Press Apps tap Save
- Enter a filename and tap Save

Once saved it can be opened by selecting Apps, scroll down and tap on file name



1. Connie measures the Pringle's container. She records a diameter of 6.8 cm and a height of 10.4 cm. According to Connie's measurements:

   a) What is the volume?

   b) What is the surface area?

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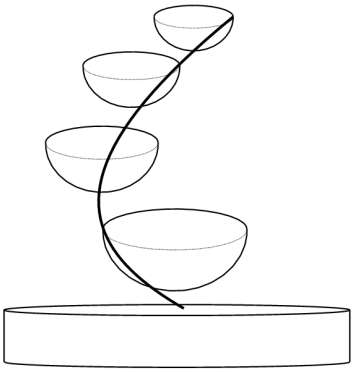
2. How much water is required to fill Diana's spa bath? It is a circular spa that is 1.2 m deep and 2.1 m across.

3. Robin is designing a water feature. It is to be made from bowls (hemispheres). The top bowl has a radius of 4 cm. Going down the next bowl has radius 5 cm, then next radius 6 cm and the bottom radius 7 cm.

   The bottom is a cylindrical shape of radius 15 cm radius and the water must be at least 4 cm deep for the pump to work.

   What is the minimum volume of water required for the cascade to work?



4. A balloon is being inflated. Assuming it is a sphere,

   a) When the radius is 4.5 cm

      (i) What is the surface area?

      (ii) What is the volume?

   b) When the radius has doubled to 9 cm

      (i) What is the surface area?

      (ii) What is the volume?

   c) How much thinner will the balloon material be?

   d) What is the ratio of the volumes of the balloon in part b) to part a)?

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Learning notes

The spreadsheet is not the only nor necessarily the most efficient way of doing these problems. It is however a further support for using formulas in context.

The distance across a circular object is the diameter.
The radius is the distance from the centre to the outside.

Press Shift 3 to enter π.

Formulae:

Shape	Surface area	Volume
Cylinder	$2\pi rh + 2\pi r^2$	$\pi r^2 h$
Sphere	$4\pi r^2$	$\frac{4}{3}\pi r^3$

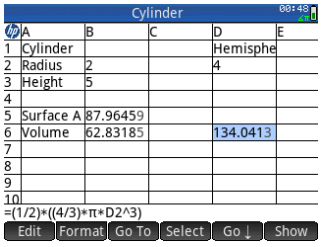
Q3 the calculation for volume needs to be done several times and the results summed. Set up the formula in the spreadsheet $\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$. As you get answers copy and paste the values to new cells and then create a formula to add them up.

Add a formula for volume of a hemisphere

- Duplicate the cylinder section for a hemisphere.

Use your spreadsheet

- Enter 4 for the radius for the first bowl
- Tap in the cell giving you the volume
- Enter radius for second bowl
- Copy answer and paste into Home screen or record answer
- Repeat for the other bowls and the bottom cylinder.



Q4 c) As the balloon fills the surface area increases and the material gets thinner, being spread over a larger area. Calculate the ratio of surface areas.

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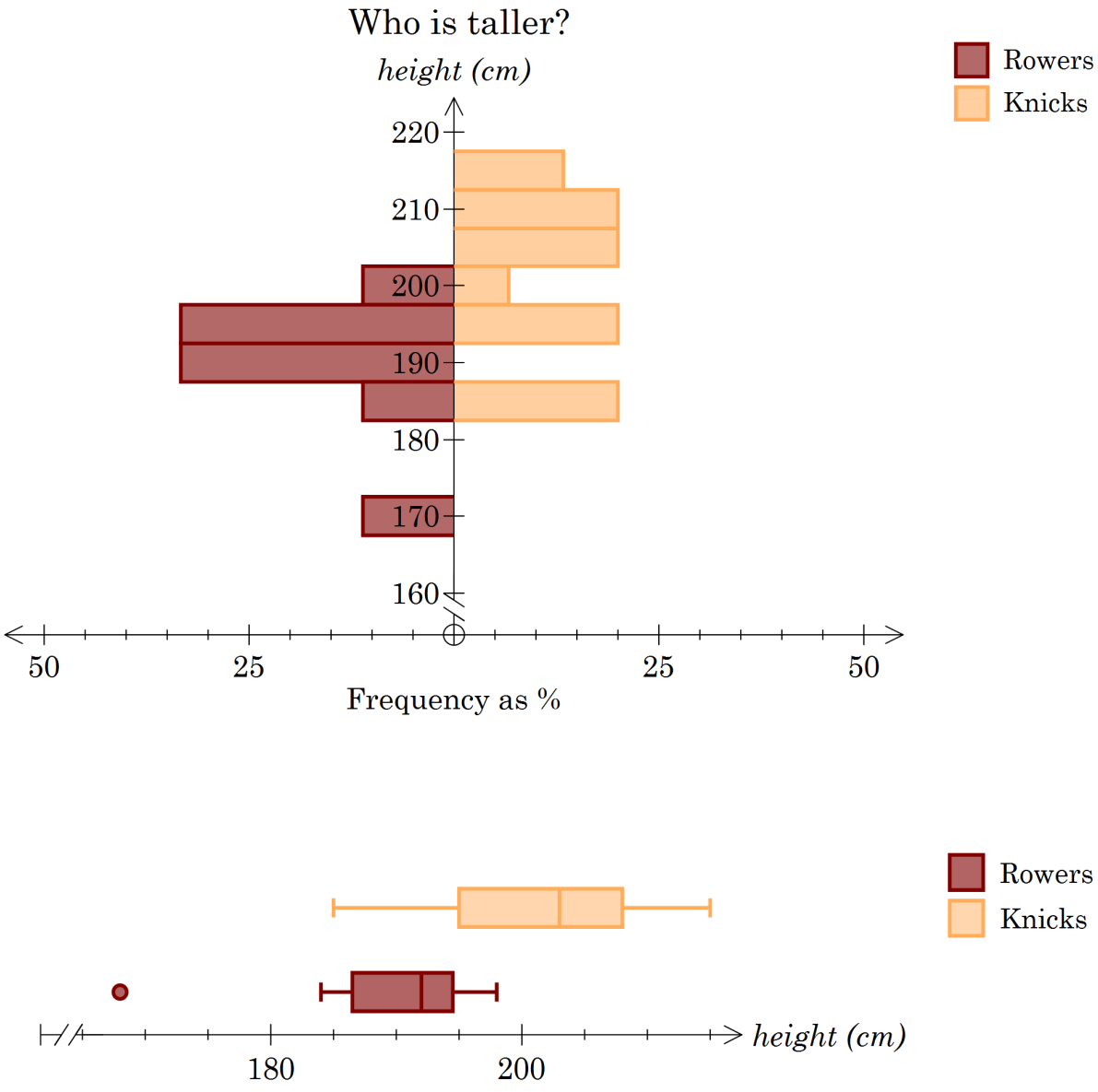
Chapter 5 Univariate data analysis

Activity	Prime apps	Key concepts
Knicks'tistics	Statistics	Calculate summary statistics
Knicks'tistics II	Statistics	Display distributions
Rowing v basketball	Statistics	Compare data sets
Reaction times	Statistics	Compare grouped data sets



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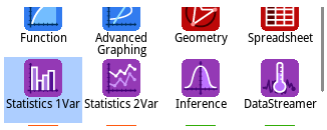
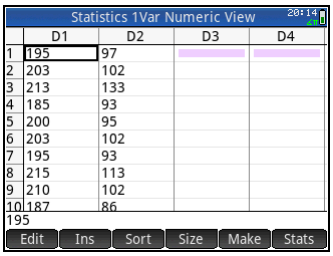
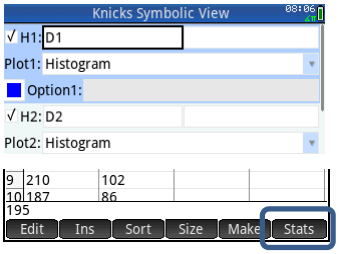
Activity 14 Knicks'tistics

Aim: Determine statistics from a list of data.

Table 1 is a list of the heights (cm) and weights (kg) of the New York Knicks Basketball Players (as at Jan 2011).

Calculate summary statistics

Open Statistics app • Press Apps and select Statistics 1Var	
Enter data: • Tap in the cell for the first element in D1. • Enter 195 and press Enter • Repeat for the remaining data. The screenshot shows heights in D1 and weights in D2	
Calculate statistics • Press Symb • Enter D1 in H1 and D2 in H2 as shown • Press Num • Tap STATS on menu line	

H (cm)	W (Kg)
195	97
203	102
213	133
187	83
185	93
200	95
203	102
195	93
215	113
210	102
187	86
208	108
208	111
197	99
205	102

Table 1

1. Complete the table for the Knicks statistics

	Height	Weight
Mean		
Median		
Maximum		
Range		
Inter-quartile range		
Standard deviation		

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2. If the players were to lie down in a straight line head to toe would they reach the 28 m from one end of a basketball court to the other? Justify your answer.

3. Does the team weigh more than 1 tonne? Justify your answer.

4. How many players on the team's roster?
Where is this value shown in the stat calculation window?

5. You are added to the roster.
   a) Add your height and weight to the lists and complete the table.

	Height	Weight
Mean		
Median		
Maximum		
Range		
Inter-quartile range		
Standard deviation		

   b) Which of the above statistics is most affected by your height and weight being included?

Save your data for later use. (see Learning notes)

EXTENSION

Gather stats on a team you are part of, or support, and complete the table.

	Height	Weight
Mean		
Median		
Range		
Inter-quartile range		
Standard deviation		

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Learning Notes

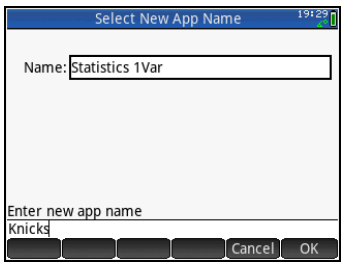
Symbols used by PRIME and their meaning.

PRIME symbol	Description
n	How many in the list
Min	Minimum value in list
Q1	First Quartile
Med	Median
Q3	Third Quartile
Max	Maximum
ΣX	Sum of the numbers in the list
$\hat{x}$	Mean or average
σX	Standard deviation

The Range is the difference between the maximum value (maxX) and the minimum value (minX)
The Inter-quartile range is the range for the middle half (Q3 – Q1).

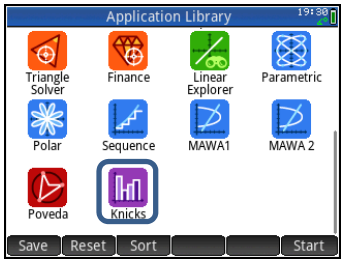
Save data

- Press Apps
- Press **Save** in menu bar
- Name the file
- Tap OK



Open file

- Press Apps
- Scroll down to the file you named above and tap the icon.
  It will be the last app as the most recent saved file.



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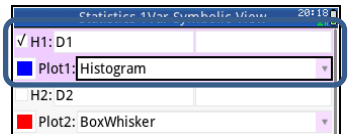
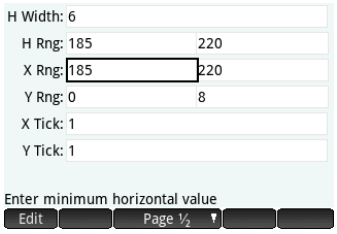
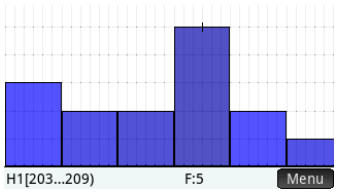
Activity 15 Knicks'tistics II

Aim: Display data using histograms and box plots.

It is assumed you have the data entered from Activity 15 *Knicks'tistics*.

Draw Histogram

Open the file saved from the last activity **Set up the graph** • Tap Symb Setup • In H1 enter D1 to select the data in list D1 • Select Histogram for Plot1	
Draw the graph • Press Shift Plot Setup to set up parameters for the graph • Press Plot	
Collect frequency table data With the graph window active • Tap Menu and Trace • Press the left and right arrows The screenshot shows there are 5 heights in the range 203 to 209 (not including 209)	

1. Complete the following frequency tables. (Use your graph and trace to get the values)

Heights: interval of 6 cm beginning at 185 cm
(H Rng: 185, H Width: 6)

Interval		Frequency
Start	End	
185	190	
191	196	

Weights : interval of 5 kg

Interval		Frequency
Start	End	
80	84	
85	89	
90		
95		
100		
105		
110		
> 115		

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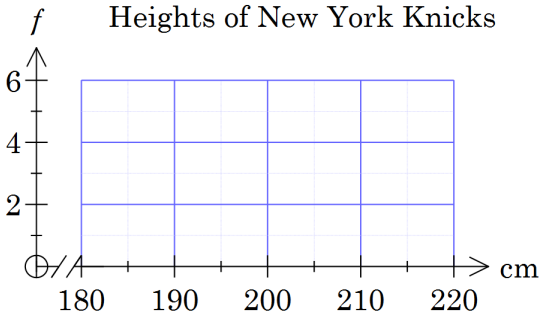
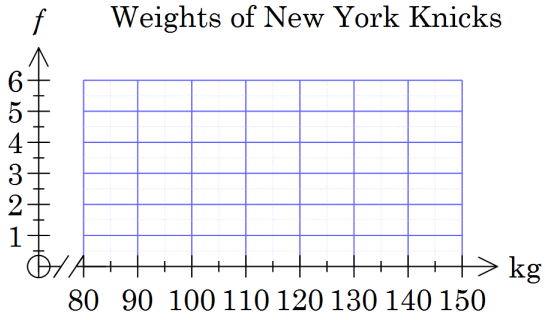
2. Draw histograms for height and weight based on your frequency tables in Q1.





3. Comment on the data. The following is a good starting point.

a) Do any values (players) stand out as being different?

b) Predict the middle height and weight by looking at the graphs.

c) Where are most of the players in terms of height and weight?

d) Describe the spread of the data. Is it clumped together, spread out or … ?

EXTENSION:

4. How is the graph affected by:

a) changing the interval (H Width:)

b) changing the starting value? (X Rng:)

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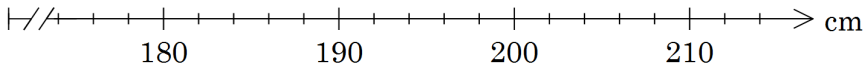
5. Draw box plots for height and weight.
Remember to use [Analysis | Trace] to get accurate values to plot.

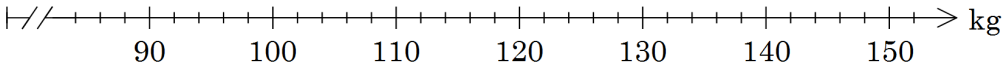
a)

Box Plot: Heights of NY Knicks roster



b)

Box Plot: Weights of NY Knicks roster

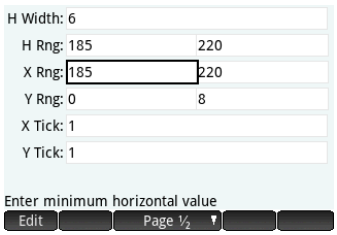


Learning notes

Scaling graph

Draw the graph

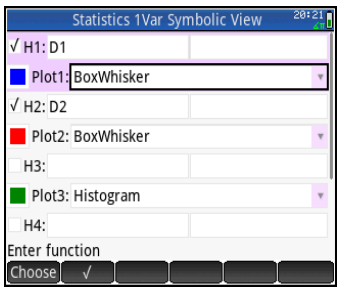
- Press Shift Plot Setup to set up parameters for the graph
  OR if you are unsure you can just plot (press Plot) and then pinch and zoom
  OR press Window and select Autoscale



Q5

Draw Box plots

- Tap Symb Setup
- Select Box Whisker
- Press Plot



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Activity 16 Rowers v Knicks

Aim: Compare data sets using graphs.

Box plots are useful for comparing data sets.
In this activity you will compare the New York Knicks professional basketball team with an Olympic men's rowing eight.

New York Knicks	
Height (cm)	Weight (kg)
195	97
203	102
213	133
187	83
185	93
200	95
203	102
195	93
215	113
210	102
187	86
208	108
208	111
197	99
205	102

Men's Rowing 8	
Height (cm)	Weight (kg)
184	87
192	94
189	86
198	94
193	92
192	87
195	93
194	90
168	55

(The last person is the cox)

1.

a) Calculate summary statistics for the heights of both groups and fill in the table. (see Learning notes for instructions)

	Heights in cm	
	Basketballers	Rowers
Mean		
Median		
Maximum		
Range		
Inter-quartile range		
Standard deviation		

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b) For each statistic listed in your table comment on what it suggests about the groups. E.g. a higher mean suggests the group is taller.

c) Summarise the results from part b).

2. Calculate summary statistics for the weights of both groups.

a) Fill in the table.

	Weights in kg	
	Basketballers	Rowers
Mean		
Median		
Maximum		
Range		
Inter-quartile range		
Standard deviation		

b) For each statistic listed in your table comment on what it suggests about the groups. E.g. a higher mean suggests the group is heavier.

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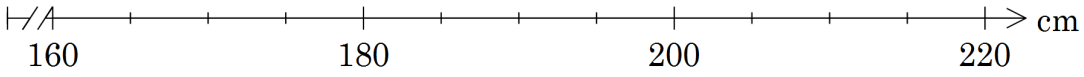
c) Summarise the results from part b).

3. Compare the groups using box plots on the axes below.

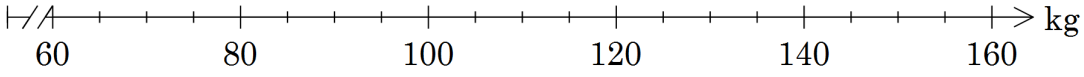
a) Draw box plots for the heights of both groups.
Use [Analysis | Trace] to get accurate values to plot.

Height boxplot: rowers & basketballers



b) Draw box plots for the weights of both groups.

Weight boxplot: rowers & basketballers



4. For comparing data discuss the relative strengths of statistics and box plots. I.e. which is more convincing to say that one group is more or less than another or to describe how the data is distributed.

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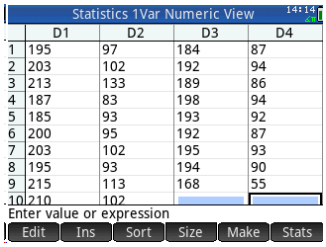
Learning notes

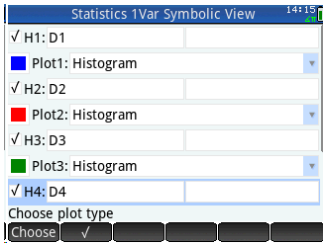
Enter the data

- Open Knicks file from earlier activity
  Press Apps and select file
- Add rowers heights and weights
  Screenshot shows basketballers' heights in D1, weights in D2, the rowers' heights in D3 and weights in D4

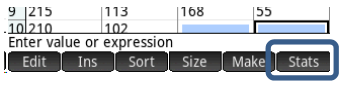


Calculate statistics

- Press Symb Setup to set up
- Assign each list to a summary variable
  (Plot doesn't matter for calculating summary statistics)



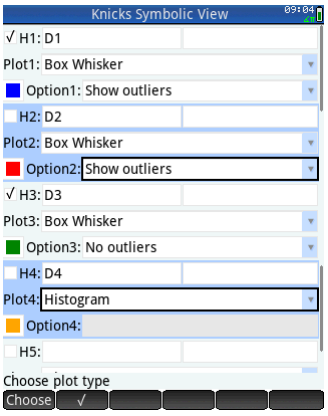
- Press Num Setup
- Tap Stats



In the previous Activity you drew graphs. To draw more than one graph at the same time turn on multiple Statgraphs.

Draw multiple boxplots on the same graph

- Press Symb Setup to set up plots
- The settings in the screenshot draw two box plots, of lists D1 and D3. This is comparing the heights of the basketballers (D1) with the heights of rowers (D3)



Draw graphs

- Press Plot Setup
- Pinch and pull to adjust view OR
- Press View Copy and toggle to AUTOSCALE
- Use trace to get accurate values to draw on paper OR
- Set parameters in Plot settings Shift Plot Setup

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Activity 17 Reaction times

Aim: Display grouped data and calculate summary statistics.

The table summarises reaction times for a data sample downloaded from the Census At School website.

Reaction Time (s)	Frequency	
Class Interval	Dominant hand	Non-dominant hand
0.2 - 0.29	16	7
0.3 - 0.39	80	55
0.4 - 0.49	59	77
0.5 - 0.59	23	31
0.6 - 0.69	7	8
0.7 - 0.79	2	2

1. Which estimate is more reasonable for the mean reaction time of the dominant hand? 0.3, 0.4, 0.5

2. Which estimate is more reasonable for the inter-quartile range of the reaction time of the dominant hand? 0.05, 0.1, 0.2, 0.4

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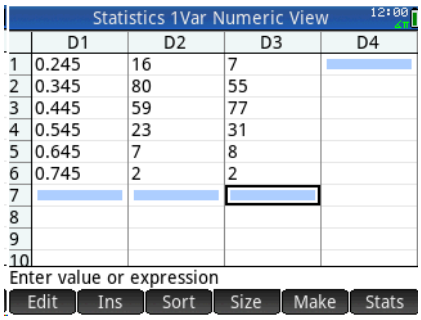
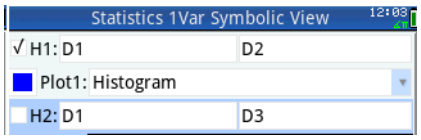
In order to calculate statistics from this data it is necessary to make assumptions about the data. Assume that all values in an interval are the same and equal (on average) to the middle of the interval.

Enter the data • Press Apps open Statistics 1 VAR app • To clear existing data: press Shift Esc and select 3All • In D1 enter the mid-point of each interval • In D2 enter the frequencies for dominant hand • In D3 enter the frequencies for non-dominant hand	
Calculate statistics • Press Symb to set up H1 and H2 as shown • Tap Stats read statistics • When finished Tap OK	
• Press Num • Tap Stats	

3. Complete the table by reading the values from your Prime calculations.

	Dominant hand	Non-dominant hand
a) Mean		
b) Minimum		
c) Lower Quartile		
d) Median		
e) Upper Quartile		
f) Maximum		
g) Range		
h) Inter-quartile range		
i) Standard deviation		

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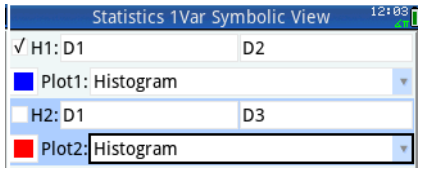
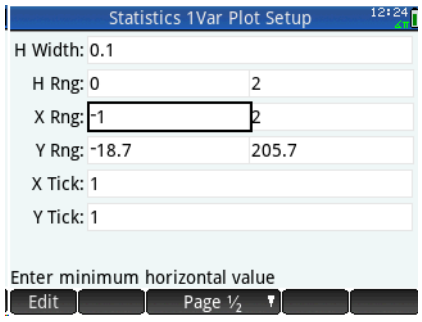
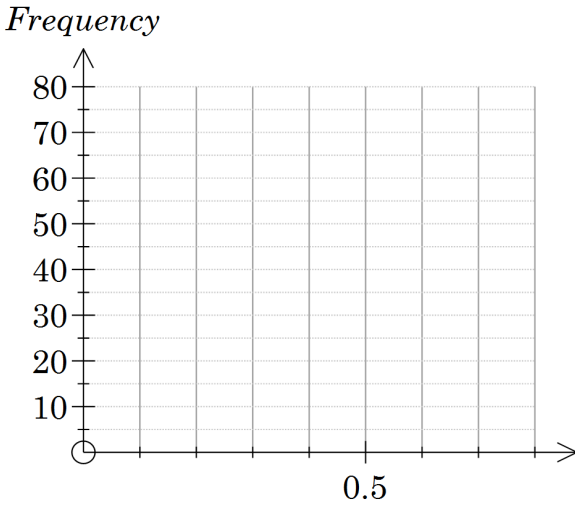
4. Draw histograms for the reaction times on your Prime.

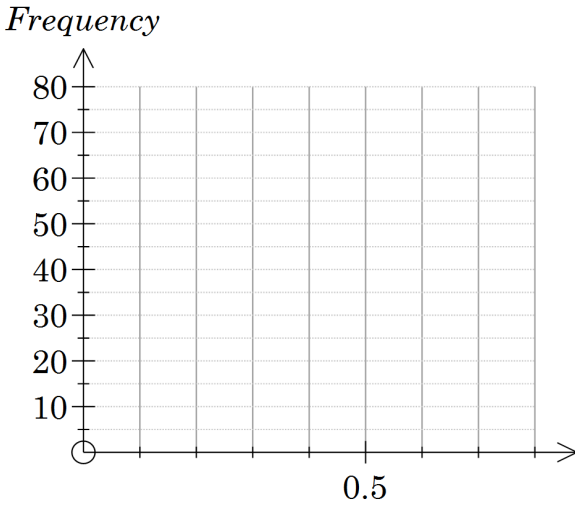
Set up to draw graph • Press Symb Setup • Ensure Plot is set to Histogram	
• Press Shift Plot Setup and set plot parameters as shown • Press Plot Setup Note for non-dominant hand: press Symb Setup and check D1 D3 and uncheck D1 D2 • Use Trace to get accurate values when plotting you graph on paper	

a) Copy the plots to the grids below

Dominant hand reaction time



Non-dominant hand reaction time



b) Comment upon the differences between the reaction times of dominant and non-dominant hands according to the graphs.

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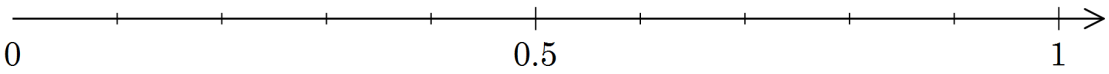
5. Draw box and whisker plots for the reaction times on your Prime.

   a) Copy the graphs to the following grid.

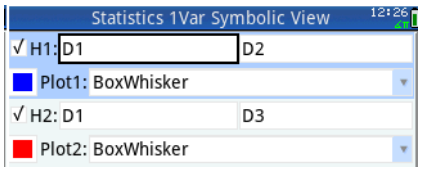


   b) Comment upon the differences between the reaction times of dominant and non-dominant hands according to the box plots.

6. In this example which type of graph is more useful for comparing the two data sets?

Learning notes

Q5

Draw multiple boxplots on the same graph • Press Symb Setup • Check both lists • Change graph type to Box Whisker • Press View Copy and adjust view window as required	
Draw graphs • Press Plot Setup	

This data was obtained from a sample from the Census at schools website http://www.abs.gov.au/censusatschool. There are a lot of statistics questions that can be asked based on the data available on this site. Many countries have such sites e.g. http://www.censusatschool.org.uk/

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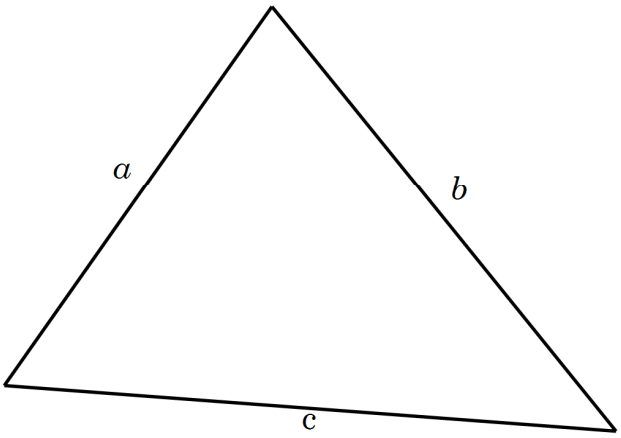
Chapter 6 Applications of trigonometry

Activity	Prime apps	Key concepts
Right-angled triangles	Triangle Solver	Solve problems involving right triangles
Right-angled triangles II	Solve	Algebraic methods for solving right triangles
Window dressing	Triangle Solver	Solve non-right triangles

Heron's rule



$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$

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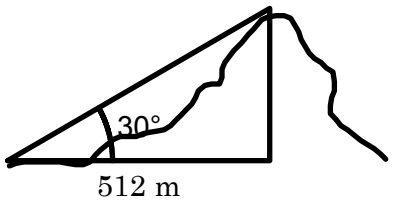
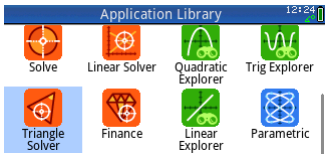
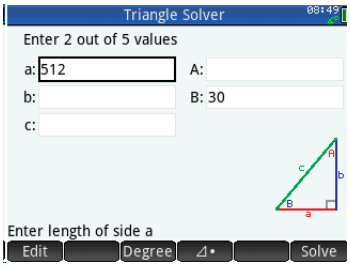
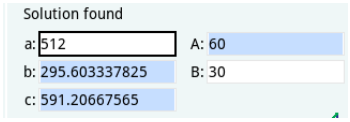
Activity 18 Right-angled triangles

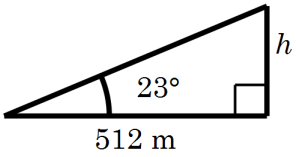
Aim: Solve problems involving right triangles.

Hiker Jim is resting in his climb of Mt Magnificent. From his map he knows he is 512 m from the summit and measures the angle to the summit as 30°. How much higher is the summit?



Open Triangle Solver App • Press Apps Info • Tap Triangle Solver App • Press Shift Esc Clear to clear	
Set the measurements • Tap right angled triangle from menu bar • Enter the values as per diagram above	
• Tap Solve to calculate values of sides and angles • Read the required value	

Write out a full solution

Working	Explanation for step in working
	Labelled diagram Include all given information and label the unknown with a symbol.
$\tan 30° = \frac{h}{512}$	Write an equation (if required)
$h = 295.6033$	Record the answer (from your calculator)
The mountain is 296 m high (to the nearest metre)	Round the answer and include the units. For a word problem answer with a sentence.

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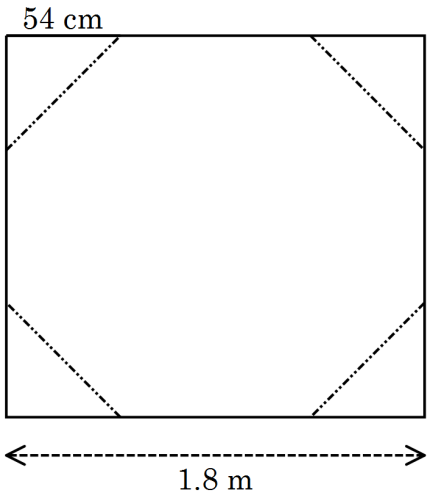
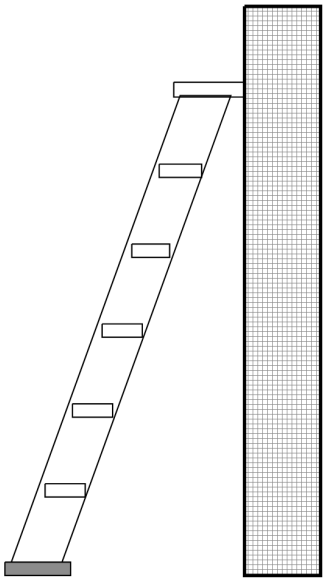
Write full solutions for the following problems.

1. The corners of a square piece of board are to be cut off to make an octagonal table. The square has sides 1.8 metres long. The triangles are cut 54 cm from the corner at an angle of 45 °. How long are the cuts?



2. What angle does the 2 metre ladder make with the ground when the base of the ladder is 0.6 metres from a vertical wall?



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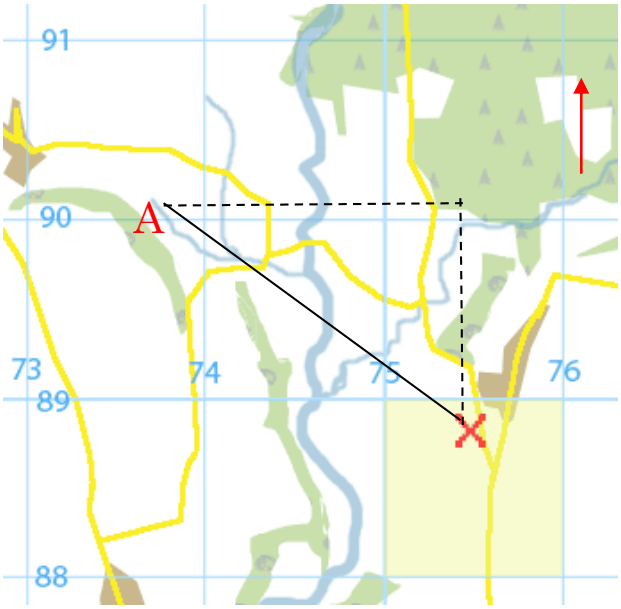
3. Point X lies 256 m from point A and 31˚ South of East from A.

How far East and how far South of A is point X?

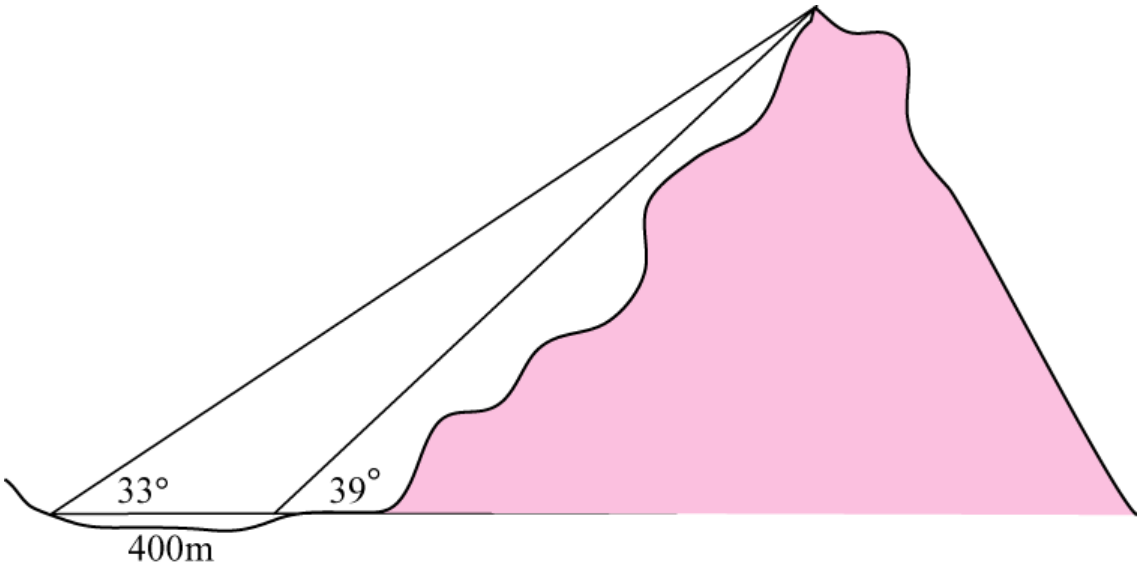


.

EXTENSION

4. Tom loves walking and is preparing to climb a mountain.

Tom measures the angle of elevation to the top of the mountain at 33°. 400 m closer to the mountain Tom measures the angle of elevation again. It is now 39°.



How high is the mountain?

Note: You are not expected to write an equation.

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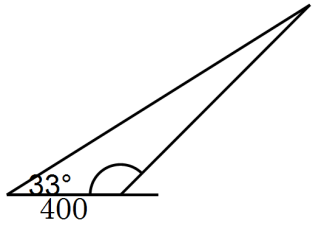
Learning Notes

Your teacher may want you to be using an algebraic approach, i.e. writing an equation and then solving it. Having written an equation as in the example the answer can still be obtained from the Triangle Solver App and the solution is complete.

Q4 Use Triangle Solver app and consider the triangle



where you can work out the other base angle.

The next activity uses algebraic methods.

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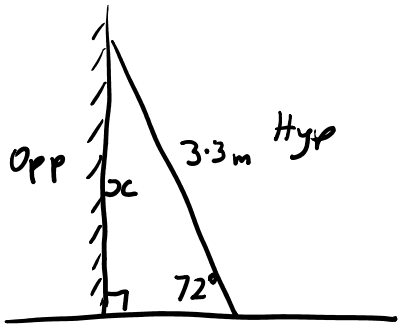
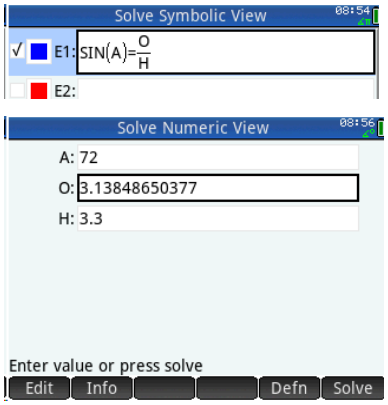
Activity 19 Right-angled triangles II

Aim: Solve right-angled triangle problems using trigonometry.

A 3.3 metre ladder is leaning up against a wall. It makes an angle of 72° with the floor. How high up the wall does the ladder reach?

Solution:

Step and explanation	Working
Draw and label a diagram.	
Choose the appropriate equation. Use the trig equation involving two measurements you know and the one you want to find.	$\sin\theta = \frac{O}{H}$
Substitute values.	$\sin 72° = \frac{x}{3.3}$
Solve the equation using the Solve App	
Write answer appropriately rounded with units.	$x = 3.1$ m (1 d.p.)
Check answer is reasonable.	Close to but less than 3.3 m is reasonable

The three questions following ask you to use different methods to solve the equation. It is valuable to use each method and then decide which is quicker and easier for you to use.

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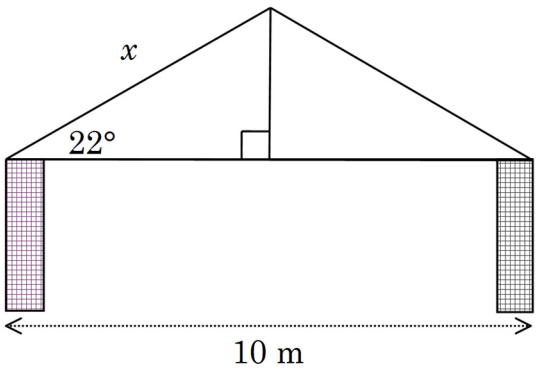
1. A carpenter is building a roof. It is to have a pitch of 22° and span walls 10.0 m apart.

a) (i) Write a full solution to calculate x, the length from the apex to the outside of the wall

.



(ii) Is a 6 m length long enough to reach from the apex of the roof to the outside of the wall?

b) How high is the apex above the wall?

2. What is the size of the smallest angle in a right-angled triangle with sides 1.49 m, 1.24 m and 82 cm?

a) Draw a diagram.

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b) You have several choices for the smallest angle: Calculate both and then write a full solution to the problem.

3. James's boat is tethered to a point 3.1 m above the boat. The rope when fully stretched is 9.6 m long. What is the angle between the stretched rope and the horizontal?

   a) Draw a diagram

   b) Use the Solve App to solve the equation and write a full solution.

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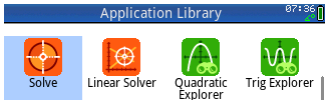
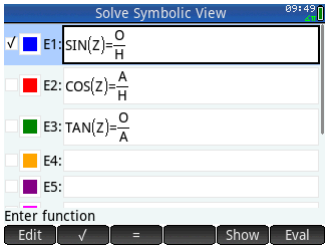
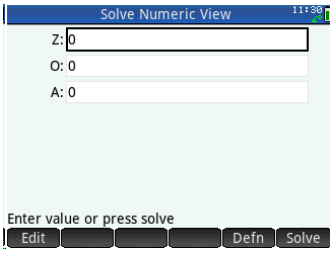
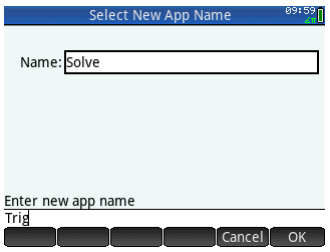
Learning Notes

It is assumed you have prior knowledge of solving right triangles using trigonometric ratios

Q3

Open Solver • Press Apps and tap Solve	
Enter the equations • In E1 press SIN ALPHA Shift 2 = ALPHA On ÷ ALPHA COS Enter • Enter the other equations as shown	
Solve equation • Check the equation you want to use • Press Num • Enter the known quantities and tap Solve	
To save the equations • Press Apps and tap Save • Enter a name and tap OK	

Triangle solver could also be used for these problems and may be simpler. This activity is however, written to support an algebraic approach using the equations:

$$\sin\theta = \frac{O}{H}$$

$$\cos\theta = \frac{A}{H}$$

$$\tan\theta = \frac{O}{A}$$

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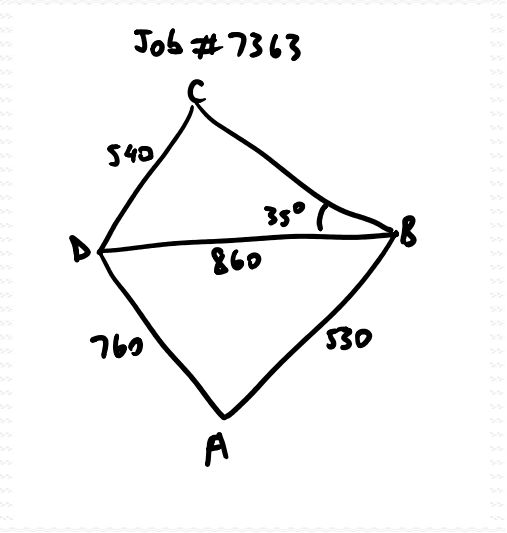
Activity 20 Window dressing

Aim: Solve non-right-angled triangles.

Norman has measured up a window for which glass is to be cut.

This is his rough sketch.

All lengths are in millimetres.



1. Use a scale diagram to measure approximate values for:

(You may use the Geometry construction Window Dressing app if your teacher has a copy or you may be able to use software such as Geogebra)

a) size of angle A (or ∠BAD)

b) size of angle ABD

c) length of diagonal AC

d) area of the whole window

e) cost of the glass given the glass costs $196.50 per square metre

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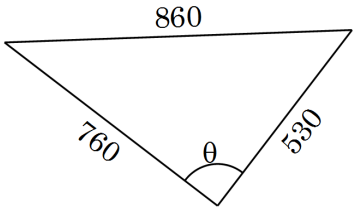
Your teacher may well want you to use trigonometric formulae in solutions of such problems.

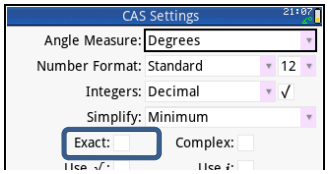
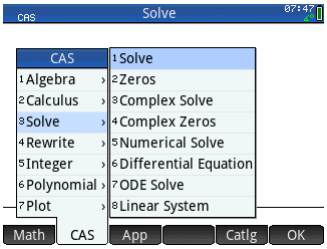
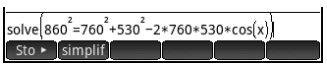
Trigonometric formulae for all triangles	
Area of a triangle	$\text{Area} = \frac{1}{2}ab\sin C$
Sine rule	$\frac{\sin A}{a} = \frac{\sin B}{b}\left(= \frac{\sin C}{c}\right)$
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$

2. With reference to this triangle:

a) Label the triangle appropriately to use the **cosine rule** to explain why $860^2 = 760^2 + 530^2 - 2\times 760\times 530\cos\theta$



b) Enter $860^2 = 760^2 + 530^2 - 2\times 760\times 530\cos\theta$ in Main and solve for θ.

Set up • Press Shift CAS Settings • Ensure settings are degrees and Exact is turned off • Press CAS Settings to open the CAS screen	
Enter the expression shown. • Press Mem and select the solve command •	
• Complete the expression as shown • Press Enter	

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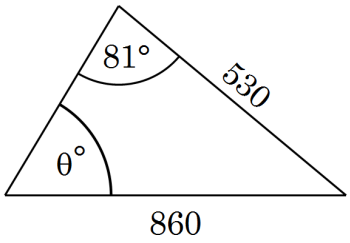
3. Label the triangle appropriately in order to use the sine rule to explain why

a) $\dfrac{\sin 81°}{860} = \dfrac{\sin \theta}{530}$

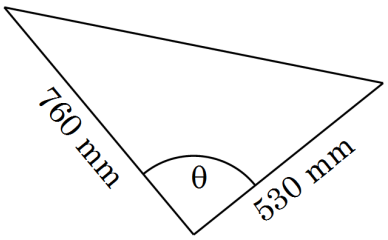


b) Enter this equation in main and solve for θ.

4. This triangle has area 1995 cm².

a) Label the triangle appropriately in order to explain why

$1995 = \dfrac{1}{2} \times 53 \times 76 \times \sin \theta$



b) Enter this equation in main and solve for θ.

Add the trigonometric formulae used in this investigation to the Solve app you saved in the previous activity *Right triangles II*.

5. Use the Solve app to do Questions 2 - 4.

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Learning notes

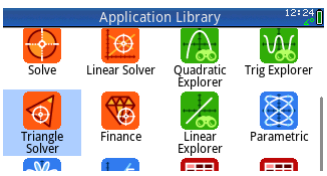
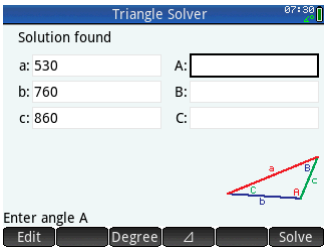
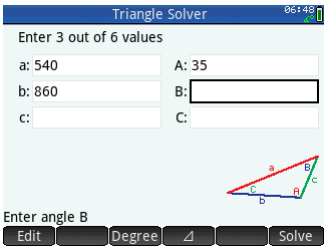
A solution is more than an answer. As a minimum a solution requires:

- a labelled diagram;
- an equation;
- substitution of the known values; and
- the answer, appropriately rounded, with units.

For solving equations you have used three methods. It is advisable to use the method that is most efficient for you for each question and this is likely to vary with the problem. The table below gives an indication of advantages and disadvantages of each method.

Method	Advantages	Disadvantages
CAS screen	• You have already written the equation.	• When finding angles PRIME output can be confusing.
Triangle Solve app	• Easy method to get the solution • Changes of values and comparison done easily	• Limited application for finding angles
Solve app	• Quick to use, just enter the appropriate values	• Limited application for finding angles

For Q1

Open Triangle Solve App • Press Apps Info • Tap Triangle Solver App • Press Shift Esc Clear to clear	
Solve ΔABD • Enter the measurements of the three sides • Tap Solve	
Solve ΔBDC • Press Shift Esc Clear to clear • Enter measurements • Tap Solve • Read the required value	

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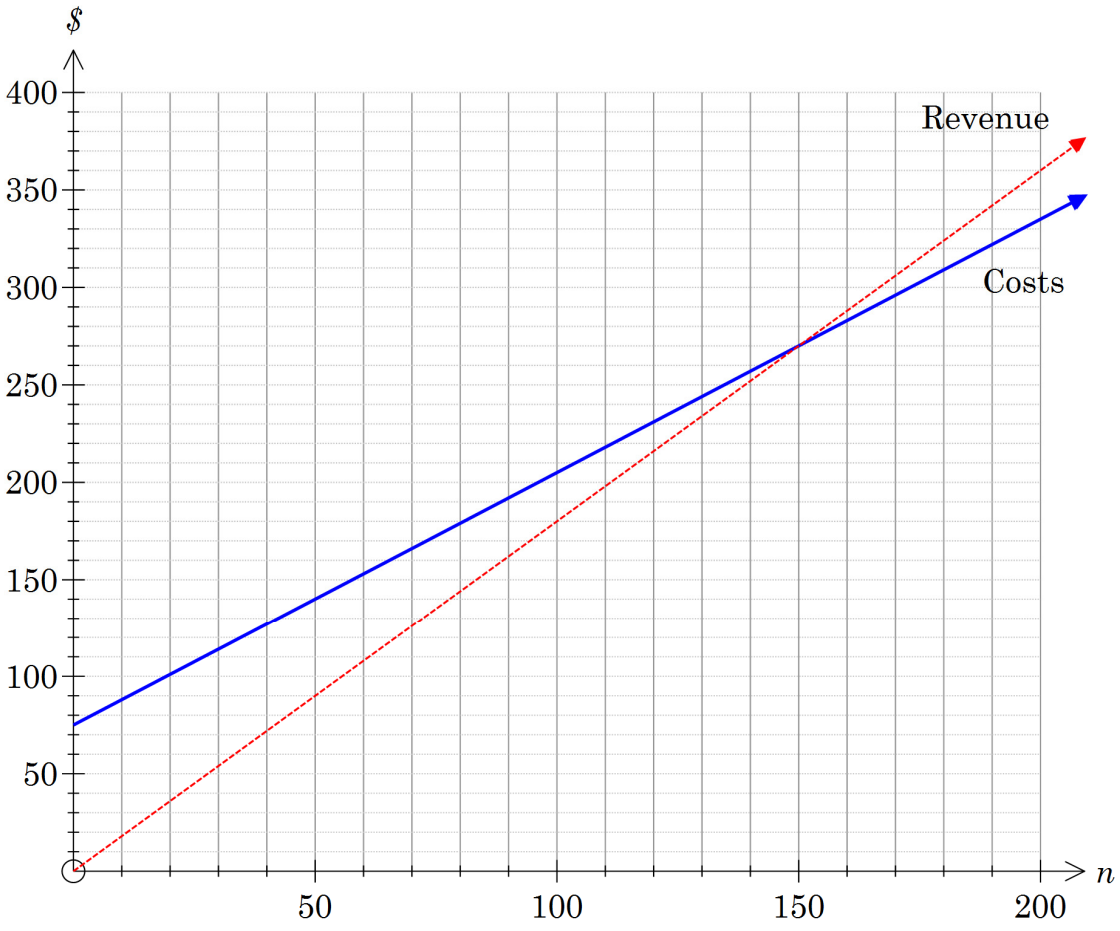
Chapter 7 Linear equations

Activity	Prime apps	Key concepts
Solving equations	CAS	Algebraic methods for solving equations.
Features of straight lines	CAS Function Graph&Table	Identify features of straight line graphs.
Deluxe taxi fares	CAS	Substitution and solving equations.
Simultaneous equations	Linear Solver	Solve systems of equations graphically
Book club	Linear Solver	Solve systems of equations in context
Income tax	Function	Piece-wise functions



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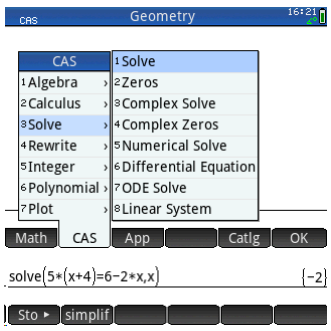
Activity 21 Solving equations

Aim: Solve equations in CAS.

To solve an equation with a single variable e.g. $5(x+4)=6-2x$

- Press CAS Settings to open CAS app
- Press Mem B tap CAS >Solve>Solve
- Enter equation
- Press Enter ≈

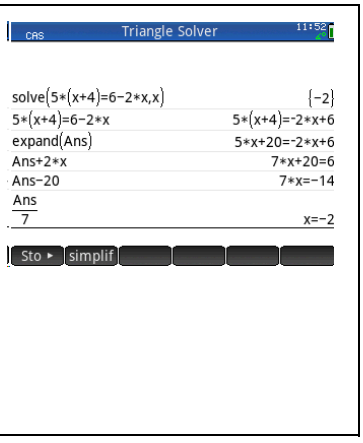


However you will sometimes be asked to solve equations algebraically, without the use of PRIME.

A full algebraic solution	The steps
$5(x+4)=6-2x$	Expand brackets
$5x+20=6-2x$	Get the x's together
$7x+20=6$	Subtract 20 from both sides
$7x=-14$	
$x=-2$	Divide both sides by 7

Use PRIME to check your algebra

- Press CAS Settings
- Enter the equation
  (or you can highlight it from a previous line, press Enter ≈ and then edit as required)
- Press Mem B tap CAS > Algebra > Expand
- Press Enter ≈
- Press (+ Ans ;), enter 2x and press Enter ≈
- Press (− Base ;), enter 20 and press Enter ≈
- Press (÷ x⁻¹ T), enter 7 and press Enter ≈



The aim with each step is make the equation simpler.
If you try something that doesn't work just go back and try again.

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Solve each equation on your PRIME without using the solve command. Then complete written **algebraic** solutions.

1. $25 - 2x = 17$

2. $2(4a - 3) = 5a + 1$

3. $\frac{2y - 4}{3} = y + 1$

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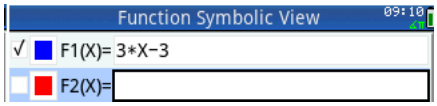
Activity 22 Features of straight lines

Aim:. Identify features of straight lines.

Setup

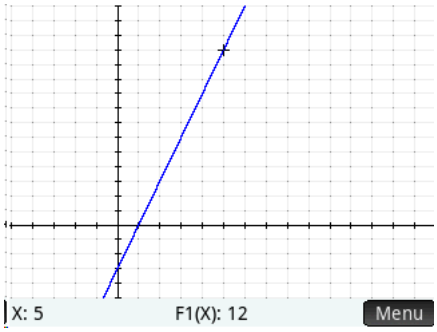
- Press Apps and Tap Function
- Enter the equation



Draw graph

- Press Plot Setup

Lines sloping up to the right have a positive slope
Lines going down to the right have a negative slope
Horizontal lines have 0 slope

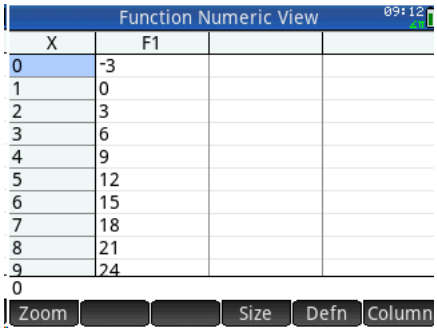


To calculate *y*-intercept

- Press Num Setup and Shift Num Setup to set up 1 step
- Press Num Setup and enter 0 Enter

Calculate *x*-intercept

- Look for 0 in the F1 column



1. Complete the table:

Equation	Slope is +ve, –ve or 0	Slope	*y*-intercept	*x*-intercept
$y = 3x - 3$	+	3	–3	1
$y = 7 - 0.3x$				
$x + y = 5$				
$2y - 3x + 6 = 0$				
$x = 4y + 3$				
$y = 3.2$				
		2.2	5	
			10	–5

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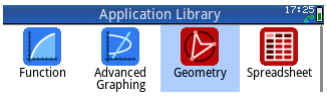
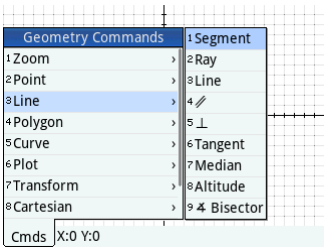
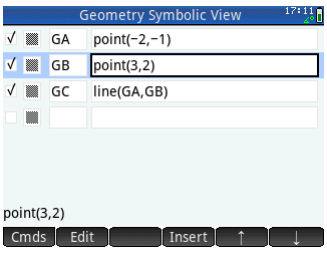
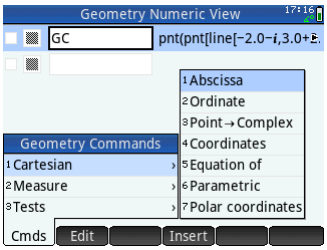
Find the equation of line between (–2, –1) and (3, 2)

Open Geometry app • Press Apps Info tap Geometry	
Draw a line • Press Plot Setup • Clear if required Shift Esc Clear • Tap Cmds > Line > Line • Tap on the screen somewhere near the first point and press Enter • Tap on somewhere near second point and press Enter to draw a line	
Constrain points • Press Symb Setup • Edit the point coordinates to the exact values specified Highlight point, tap Edit and press Enter when finished	
Read equation • Press Num Setup • Tap Cmds > Cartesian > Equation of • Enter GC, Geometry variable C which was shown in the Symbolic view • Read equation	
New pair of points • Press Symb Setup • Edit the points • Press Num Setup	

2. Complete the table.

Points	Slope	Equation of line	y-intercept
(–2, –1) and (3, 2)	0.6	$y = 0.6x + 0.2$	0.2
(1, 3) and (6, 8)			
(3.2, 1.8) and (4, –0.6)			
(2, 6) and (2, –5)			
(10, 0) and (0, 5)			

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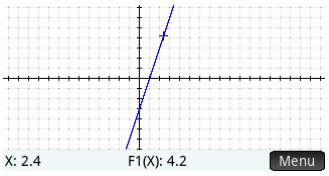
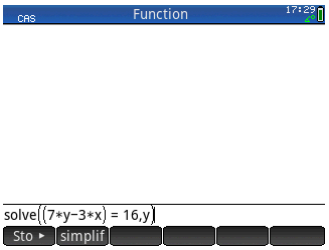
Learning notes

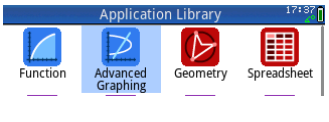
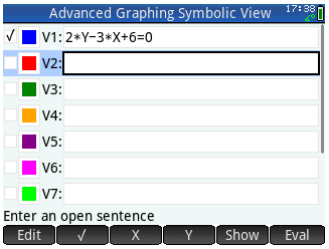
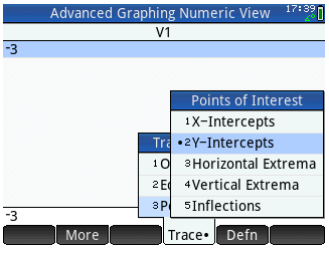
The quickest way to get an estimate for a value is to use Trace. In Function app, tap Trace• and

Trace in Function app • Tap Trace•, if not visible tap Menu to make visible • Tap on the screen close to where you want to see the coordinates, e.g. an x-intercept • You can tap again to refine the position	

Q1 For equations like $2y - 3x + 6 = 0$ you have to rearrange to plot using Function app. This can be done by…

Rearrange equation using CAS • Press CAS Settings • Select solve from the CAS menu • Enter equation • Press ? Eval and enter y • Press Enter ≈	
Plot graph • Highlight the result and press Shift View Copy to copy • Press Apps Info and select Function • Press Shift Menu Paste to paste the equation	

You can also use Advanced Graphing app to see what the graph looks like.

Plot graph • Highlight the result and copy Shift View Copy • Press Apps Info and select Function • Press Shift Menu Paste to paste the equation	
Enter equation • Use the buttons at the bottom of the screen for X and Y easy entry	
Intercepts • Press Num Setup to enter numeric view • Tap Trace• > PoI > X-Intercepts • Tap Trace• > PoI > Y-Intercepts	

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Activity 23 Deluxe Taxi fares

Aim: Evaluate expressions by substitution.

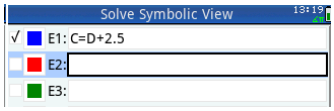
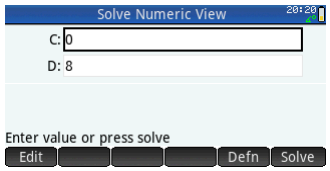
1. Deluxe Taxis: $4 per kilometre and $2.50 flagfall.

   a) Explain why a trip of 8 km will cost $34.50.

   b) What is the cost of a 12.5 km trip?

   c) How many kilometres can a passenger travel for $50?

2. The cost C for travelling a distance of d kilometres is $C = 4D + 2.5$.
   Repeat Question 1 using Solve app.

Enter the formula in Solve app • Press Apps and tap Solve • Enter the formula into E1 and press Enter	
Evaluate the unknown • Press Num • Enter the value known $D = 8$ • Press Enter • Select the unknown • Tap Solve	

3. Use the methods from Question 2 to calculate:

   a) what Ben would expect to pay for a taxi ride of:

      (i) 6.3 km

      (ii) 32.4 km?

   b) how far can Ben go for:

      (i) $10

      (ii) $78?

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4. Most taxi's will also charge a fee for waiting time. This only applies if the taxi is stationary for more than two minutes and Deluxe Taxis charge 55c per minute.
The formula now becomes $C = 4D + 2.5 + 0.55M$ where m is the number of minutes of waiting time.

   Edit the formula in Main, then use the methods from Question 2 to calculate answers and write full solutions.

   a) What would Ben expect to pay for a taxi ride of 6.3 km with 10 minutes waiting time?

   b) How far can Ben travel for $20 with a 5 minute wait?

   c) How long can Ben wait if his journey is 8.5 km and he has $50?

5. Fuel prices have rapidly increased and the taxi board has authorized a new fare schedule.

Flag fall	$4.30
Cost per km	$4.71
Wait time	$0.60 per minute

   a) Write a new formula for the cost of the taxi fare.

   b) Use Prime to calculate answers and then write full solutions:

      (i) What would Ben expect to pay for a taxi ride of 6.3 km with 10 minutes waiting time?

      (ii) How far can Ben go for $20 with a 5 minute wait?

      (iii) How long can Ben wait if his journey is 8.5 km and he has $50?

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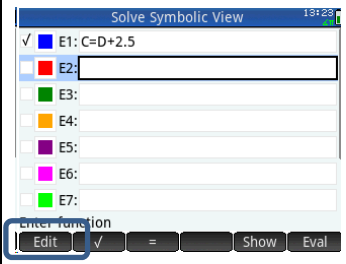
Learning notes

To Edit the formula

- If not in Solve app, press Apps and tap Solve
- Tap Edit
- Edit the formula
- Press Enter



Full solutions

Writing solutions is more than giving a number for an answer. To provide a full written solution to problems like these you should:

- Write the formula
- Substitute the known values
- Evaluate or solve the expression
- Round the answer appropriately for the problem and write the answer in a sentence.

Example 1

What would Ben expect to pay for a taxi ride of 8km?

$C = 4D + 2.5$	Write the formula
$= 4 \times 8 + 2.5$	Substitute
$= 34.5$	Evaluate
Ben's trip costs \$34.50	Write a sentence.

Example 2

How many kilometres can Ben go for \$50?

$C = 4D + 2.5$	Write the formula
$50 = 4D + 2.5$	Substitute
$D = 11.875$	Solve for d
Ben can travel 12 km to the nearest kilometre.	Write a sentence.

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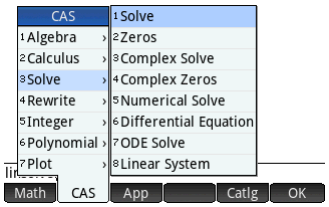
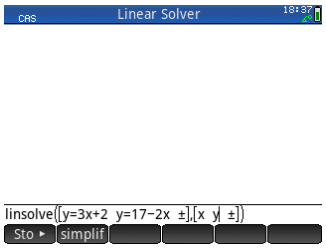
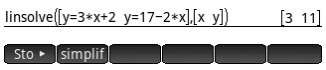
Activity 24 Simultaneous equations

Aim: Use a variety of methods to solve pairs of simultaneous equations.

In this Activity different methods for solving simultaneous equations are explored. They each have advantages depending upon the exact type of equation. After completing the activity decide what your preferred method is.

Using **CAS: Solve Linear Sys**tem

Open CAS view • Press CAS Settings • Select CAS Solve Linear System	
Enter equations • Press Shift [ ] 5 v • Enter equation • Move right (Press right arrow) • Enter second equation • Move right twice • Press Eval Shift [ ] 5 v • Enter x move right enter y	
Solve • Press Enter ≈	

Note this may take a bit of practice to become proficient entering the equations.

1. Solve the following pairs of simultaneous equations

a) $y = 4x - 6$
$3x = 2y + 7$

b) $3x + 7y = 4$
$2x - 7y = 6$

c) $5b + 1 = 2a$
$a = 2b + 8$

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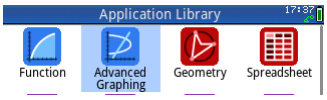
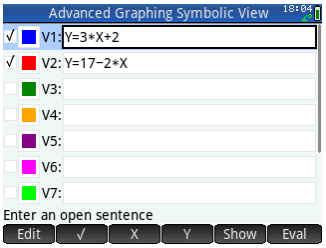
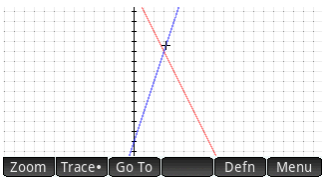
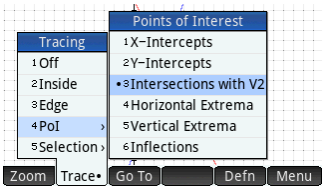
Solve simultaneous equations graphically e.g. solve $\begin{matrix} y = 3x + 2 \\ y = 17 - 2x \end{matrix}$ for x and y:

Open Advanced Graphing app • Press Apps • Tap Advanced Graphing	
Enter equations • Use the X and Y buttons to enter X and Y	
Solve graphically • Press Plot Setup You may need to use zoom to see the lines intersecting	
• Tap Menu and Trace• > PoI > Intersections with .. • Press Enter Note: You can often solve by inspection, i.e. read the coordinates off the graph.	

2. Sketch graphs for each pair of lines and state the solution to the pair of simultaneous equations.

a)

$y = 4x - 6$

$3x = 2y + 7$

b)

$3x + 7y = 4$

$2x - 7y = -9$

c)

$5y + 1 = 2x$

$x = 2y + 8$

Note: you can edit the equations or add new ones

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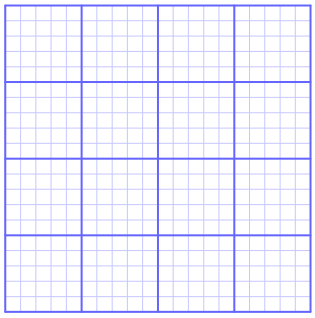
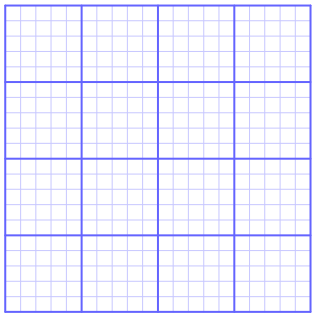
3. Solve the following systems of equations using the graphical method.

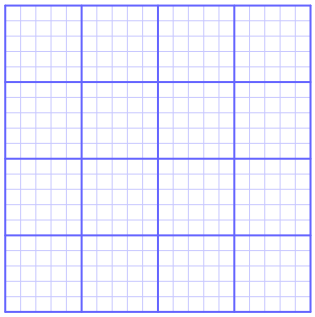
a) $y = 2x - 9$
$y = 27 - 4x$



b) $y = -4x + 14$
$y = 6x - 6$

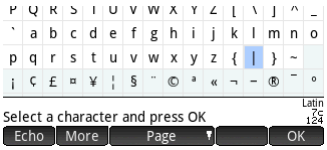


c) $x + y = 1$
$y = 2x + 3$



Solve simultaneous equations algebraically in CAS (substitution method)

You may be asked to solve problems like this (simple numbers) without Prime.
In this example an equation is of the form $y=$ so we will substitute for y

Open CAS view • Press CAS Settings	
Enter first equation	
• Press Shift Vars • Tap |	
• Enter second equation • Press Enter	

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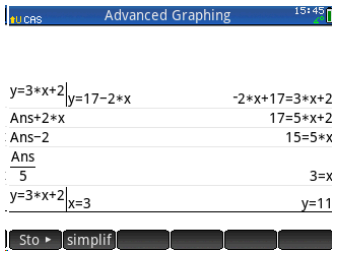
Solve for *x*

- Do the same things to both sides of the equation until *x* is isolated

Determine *y*

- Tap original equation Tap Copy and delete the second equation
- Tap | and enter x=3 (your answer from above)
- Press Enter



The solution as it would then be written on paper as

$y = 3x + 2 \quad \ldots 1$

$y = 17 - 2x \quad \ldots 2$

$-2x + 17 = 3x + 2$ substitute equation 2 into equation 1

$17 = 5x + 2$

$15 = 5x$

$x = 3$

$y = 3 \times (3) + 2 = 11$

$x = 3, y = 11$

4. Write out solutions for the following simultaneous equations.

a) $y = 4x - 6$

$x = 2y + 5$

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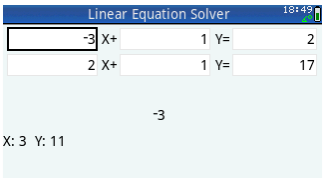
b) $x = y - 4$
$3x - 2y = -8$

c) $x = 2y + 1$
$2y = 5x - 4$

Learning notes

In this activity you have explored three methods for solving systems of linear equations, solve linear system, graphically and algebraically. Each has their advantages although you will be likely to prefer a particular approach.

You can also use Linear Solver when equations are in the form $y - 3x = 2$
$y + 2x = 17$

Open Linear Solver app	
Select 2 x 2 system	
Enter equations	

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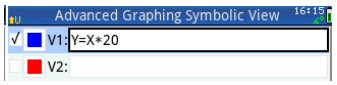
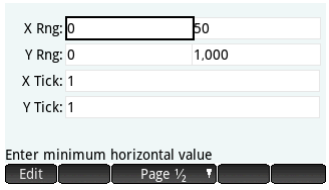
Activity 25 Book club

Aim: Use graphs to solve simultaneous equations.

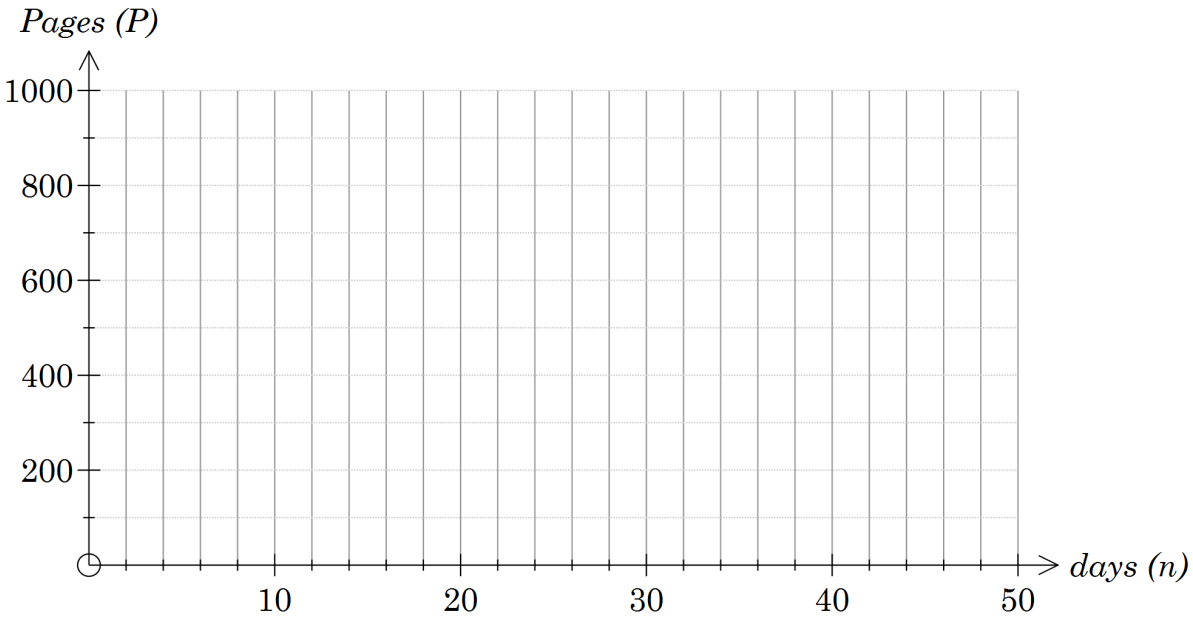
Jen and Ahjoy are friends who enjoy reading books together.
Jen reads 20 pages per day (she is highly organised).
Ahjoy starts off slowly but then reads more pages per day as the book gets exciting.

1. Write a formula for the number of pages (P) Jen has read n days after starting to read a book.

2. Draw the graph on the grid below.

Enter formula • Open Advanced Graphing • Enter the function in V1	
Adjust the view window • Press Shift Plot Setup • Set domain and range to match the grid below • You can use pinch and pull but drawing the graph on the grid may be more difficult	
Draw the graph • Tap Plot Setup	

.



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3. Ahjoy estimates his progress in reading a book with the formula $P = \frac{4n^2 - 3n + 180}{5}$.

   Enter this function as $Y = \frac{4X^2 - 3X + 180}{5}$ in Prime

   View the graph and copy to the above grid.

4. Use your graph to estimate when Jen and Ahjoy will be at the same place in the book.

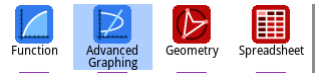
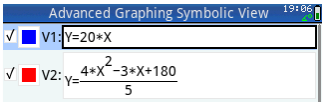
5. Their next book is 800 pages long. If they want to finish at the same time, how many days later does Ahjoy need to start reading?

6. Another of their "must read" books is a 1000 page doorstopper.

   If they want to finish at the same time, how many days earlier does Ahjoy need to start reading?

Learning Notes

Book club requires solving a pair of equations simultaneously. This is an extension of the course in that one of the equations is not a straight line. However the graphical method of solution will work whenever you are able to draw a graph.

Open Advanced Graphing app	
Enter equations • Use the X and Y buttons to enter X and Y	

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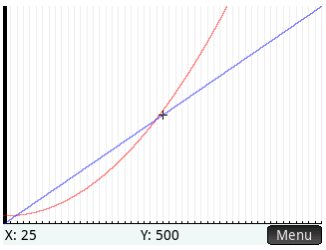
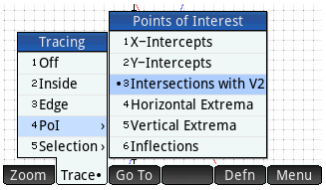
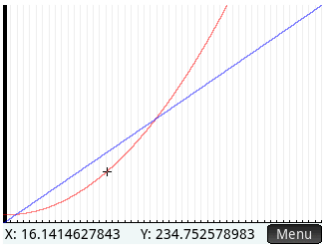
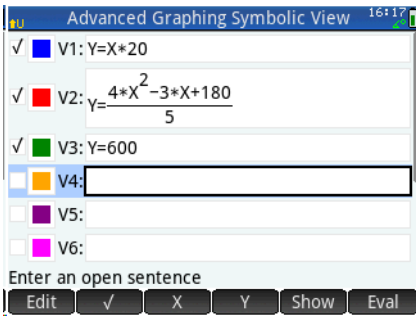
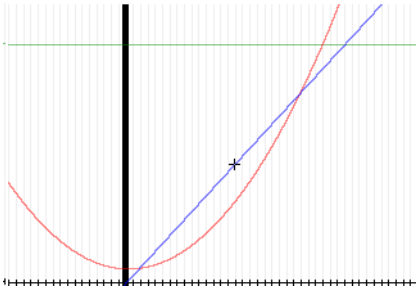
Solve graphically • Press Plot Setup You may need to use zoom to see the lines intersecting	
• Tap Menu and Trace• > PoI > Intersections with .. • Press Enter	
Note: You can often solve by inspection, i.e. read the coordinates off the graph. Trace can also be used • Trace• > Selection to choose a graph • Then tap on the screen and the cursor will move to the point on the chosen graph closest to your tap	

Q5 Use trace and look for how long before each reaches 600 pages
or
Draw another graph y3=600 and locate the intersections
or
Tap Go To and calculate x when y is 600.

Draw horizontal line • Enter a new function e.g. V3=600 • Draw the graph	
Find point of intersection • Q4 Press Plot Setup • Tap Menu • Tap Trace• >PoI>Intersection V2 • Press Enter	

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Activity 26 Income tax

Aim: Solve problems involving piecewise linear functions.

Tax rates 2013-14

Taxable income	Tax on this income
0 – $18,200	Nil
$18,201 – $37,000	19c for each $1 over $18,200
$37,001 – $80,000	$3,572 plus 32.5c for each $1 over $37,000
$80,001 – $180,000	$17,547 plus 37c for each $1 over $80,000
$180,001 and over	$54,547 plus 45c for each $1 over $180,000

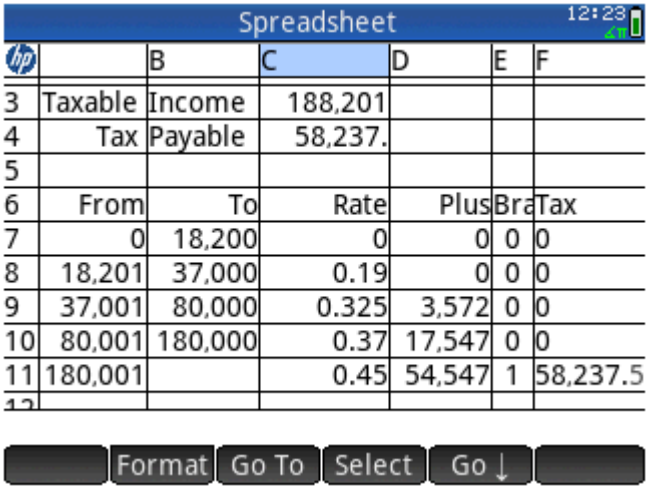
Source: http://www.ato.gov.au/Rates/Individual-income-tax-rates/

Part A: Spreadsheet

1. Use the table to calculate the tax on incomes of
   a) $14 250
   b) $18 201
   c) $91 798
   d) $32 310

2. Construct a spreadsheet to calculate the tax on any income.

See Learning notes for instructions to create the spreadsheet as shown.



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Use your spreadsheet to complete the table

Name	Taxable income	Tax payable
a) Alessia	$26 065	
b) Bruce	$126 340	
c) Christine	$76 922	
d) Dylan	$16 980	
e) Edna	$40 694	
f) Fletcher	$234 560	

Part B: Piecewise function

Prime enables us to define different rules for different inputs as the tax table requires.

3. If the taxable income is $\$x$

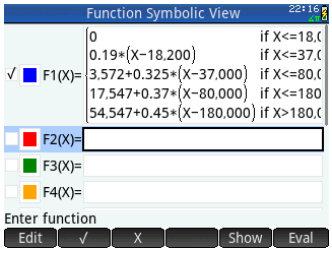
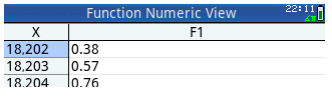
a) Explain why the expression $3572+0.352(x - 37\,000)$ gives the tax due when the taxable income is between 37 000 and 80 000.

b) Write similar expressions for the tax due on incomes between

(i) 18 200 and 37 000

(ii) 80 000 and 180 000

Define a tax function as a piecewise function

Enter the piecewise function • Open Function app • Enter PIECEWISE(0,X<18200,0.19*(X-18200),X<37000,3572+0.352*(X-37000), x<80000,17547+0.37*(X-80000),X<=180000,54547+0.49*(X-180000) ,X>180000)	
Calculate tax on 18202 • Press N • Enter 18202 (or any number) in the X column	

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c) Use your function to complete the table

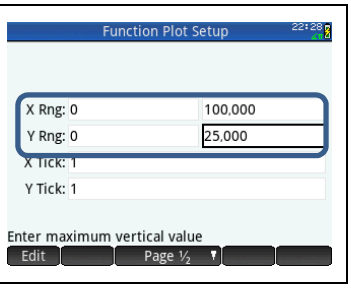
Name	Taxable income	Tax payable
(i) Gillian	$261 065	
(ii) Hadi	$18 000	
(iii) Ita	$19 100	
(iv) Jack	$20 000	
(v) Kate	$87 694	
(vi) Lam	$111 560	

4. What does the graph of tax paid against taxable income look like?

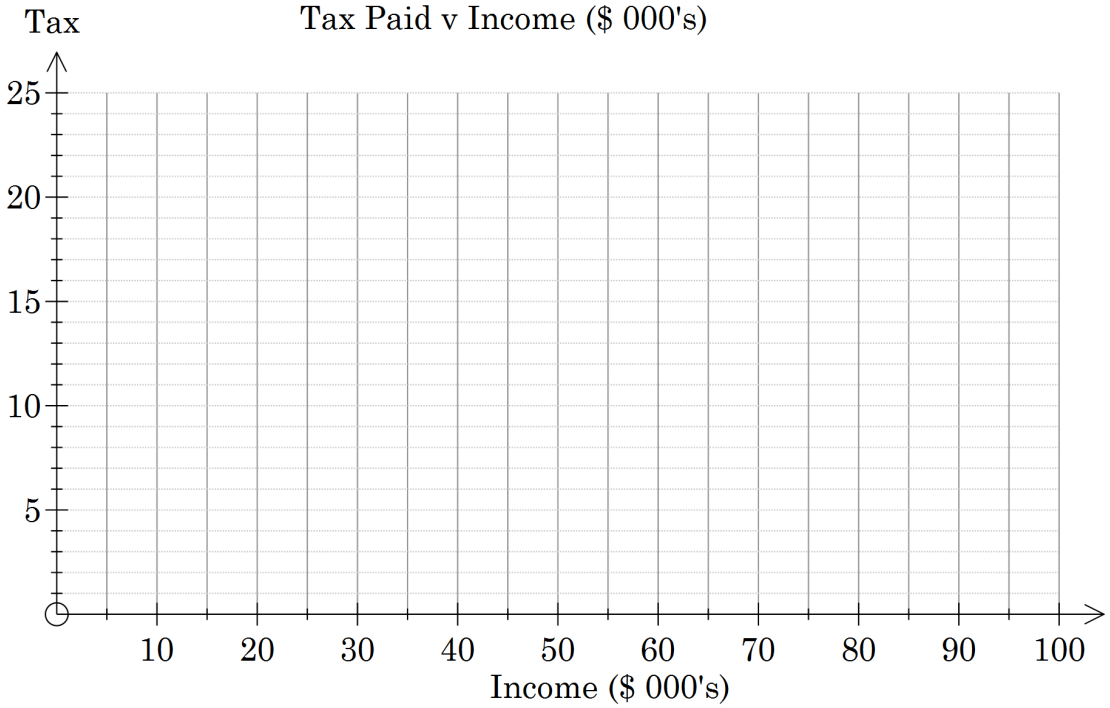
Set the view window

Consider income up to $100 000 and tax to $25 000

- Press Shift Plot Setup
- Set values
- Press Plot Setup to draw the graph



a) Transcribe the graph from Prime to the grid below.



b) At what points does the slope change?
Why?

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Learning notes

Most people pay tax each time they are paid. Tax returns are about making appropriate adjustments and for most people that means a small refund as a little more has been taken out than they need to pay.

Q2 Create the Spreadsheet. There are a few features of the spreadsheet app you need to use for these instructions: adjust column widths, align cells, select multiple cells, copy and paste formulae and the IFTE command.
It is assumed you are comfortable with entering labels and formulae into Spreadsheet app.
An important idea is to choose the appropriate rule (tax bracket). In the spreadsheet this is through the use of IFTE statements.

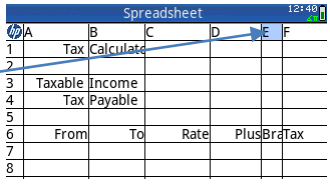
Open Spreadsheet app

- Tap Apps Info and select Spreadsheet

Enter labels

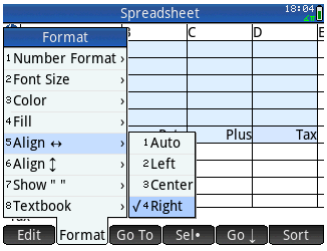
Adjust Column width e.g. make column E narrower

- Tap at the top of the Column E
- Tap Format > Column↔
- Set to 20



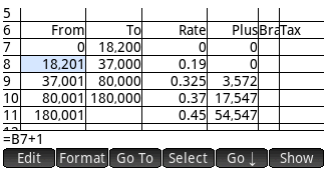
Right justify labels

- Select all cells
  Tap in A1, hold until Select changes to Sel• drag to select block of cells
- Tap Format > Align↔ > Right



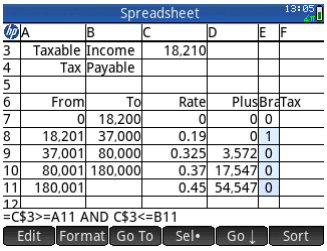
Enter data from Tax Table

- In cells A7 to C10 enter the numbers as shown in the tax table (start of the activity)
- You might like to right justify these cells and use formulae for cells A8 to A11



Determine tax bracket

- Tap in C3 and enter 18 210
  This is in the $18 200 to 37 000 bracket
- Tap in E7, enter =C$3<=B7
- Tap in E8 and enter
  =C$3>=A8 AND C$3<=B8
  This will have the value 1 (true) when Taxable income is in the 18 201 to 37 000 bracket, otherwise it is 0 (false)



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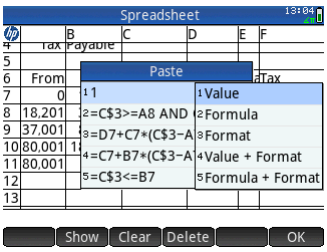
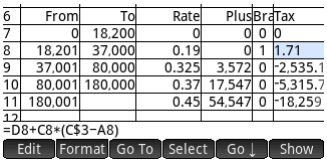
Copy the formula

- Copy the formula to cells D9 to D11
- Tap in D7, press Shift View Copy to copy the formula
- Tap in D7, hold until Select changes to Sel•
- Drag down to D10
- Press Shift Menu Paste select the 0.19 and 2Formula or 5Formula+Format



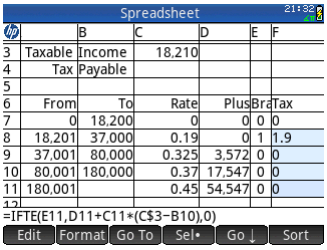
Calculate tax for tax bracket

- In cell F7 and enter =D7+C7*(C$3–A7)
- In cell F8 enter =D8+D7*(C$3–B7)
- Copy formula in F8 to cells F8:F11



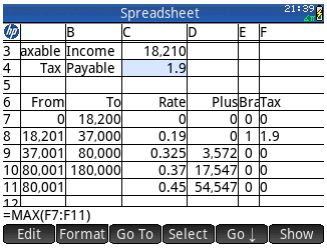
Modify formulas

- Tap in F7, edit the formula to **=IFTE(E7,**D7+C7*(C$3–A7)**,0)**
- Copy the formula to cells F8:F11
  Now only the tax bracket which fits the taxable income has a value other than 0.



Display taxable income

- Tap in C4, enter the formula =MAX(F7:F11)
  This is the maximum value in cells F7 to F11, i.e. the correct tax bracket and tax. Another function that could be used is the SUM function, i.e. =SUM(D7:D10)



Q3 c) You may like to explore the percentage of income that is paid in tax. You may have heard people on high incomes complaining about the amount of tax they need to pay. For example Kate pays $20 393.78 tax on a taxable income of $87 694. This is about 23%, much less than the marginal rate of 37%.

Yet another approach is to write a program to calculate the tax.

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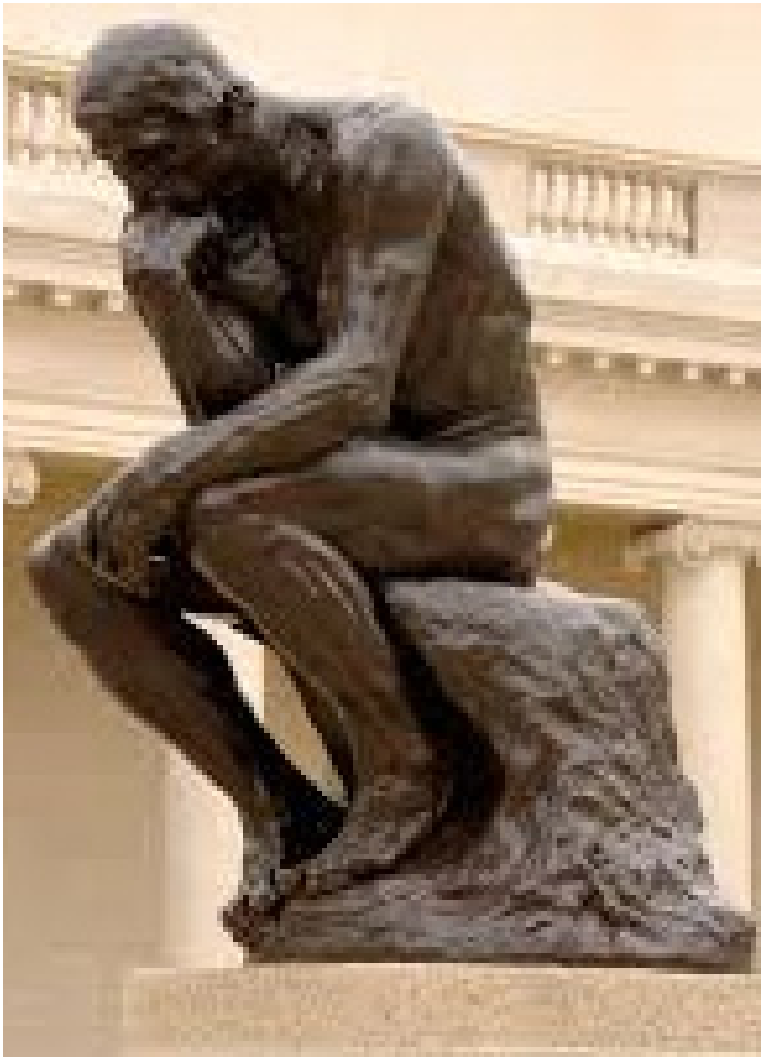
Chapter 8 Problems

The Activities in this chapter go beyond the standard course. They are useful in extending your Prime skills and are interesting applications or extensions of the course content. Doing these will help you apply your knowledge to new situations and will be useful preparation for Units 3 and 4.

Activity	Prime apps	Key concepts
Phone costs	Main	Use and understand function notation, solve linear equations
Moderating marks	Statistics	Adjusting school marks to match the centre and spread of the exam marks



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Activity 27 Phone costs

Aim: Use and interpret function notation.

Suzie's pre-paid account with *FourMobile* has $250 value. The table below shows how Suzie is charged for her calls.

Local rates per minute (?)	
Call rate per minute or part thereof	$ 0.89
Flagfall rate per call	$ 0.39

1. Study Suzie's call records listed in the following table.

Date	Time	Phone Number	Duration	Call minutes
1/3/12	4:17		6:54	7
1/3/12	4:24		18:25	19
1/3/12	5:11		0:05	1
1/3/12	5:11		0:42	1
2/3/12	5:12		12:15	13
2/3/12	6:12		2:00	2
4/3/12	3:59		17:01	18
4/3/12	7:05		1:12	2
4/3/12	7:29		21:34	22

   a) How many calls has Suzie made?

   b) What is the total number of call minutes Suzie will be charged for?

   c) What is the cost of Suzie's calls (including flag fall and rate per minute costs)?

   d) How much of the $250 credit does Suzie have left?

   The credit remaining on this $250 plan is a function of the number of calls, N and the number of call minutes, M.

$$c(N,M) = 250 - 0.39N - 0.89M\,.$$

For example after 20 calls and 100 call minutes the remaining credit is $c(20,100) = 250 - 0.39\times 20 - 0.89\times 100 = \$153.20$.

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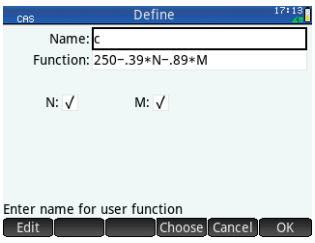
2. Complete the table.

	Number of calls	Call minutes	Credit remaining ($)
$c(10,250)$			
$c(50,150)$			
	72	175	
$c(32,\quad)$		220	
$c(\quad, 200)$			\$56.40

3. What is the maximum number of calls that could have been made if there were 250 call minutes?

Define the function in Prime • Press CAS Settings to open CAS • Press Shift $xt\theta n$ Define D then ALPHA alpha $\sqrt{}$ Units c and press Enter ≈ to call the function c • Use the keyboard to enter 250–0.39N–0.89M for the expression. (Use Shift ALPHA alpha $( )$ N to enter N) and tap Enter ≈ (Use capital letters for variables in Prime functions) • Tap OK	
Evaluate function • Press and enter the values given E.g. enter c(10,20) to find the credit after 10 calls and 20 call minutes	

4. Use your Prime function to answer the following questions.

a) What is the credit remaining after 72 calls and 240 call minutes?

b) What is the credit remaining after 16 calls and 250 call minutes?

c) Suzie checks her balance and notices it is \$45.26 and that she has made 64 calls. How many call minutes has Suzie made?

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5. Record the Prime output for the following inputs:

a) $c(10, m)$

b) $c(10, \text{mins})$

c) $c(x, y)$

d) $c(10, 2m)$

e) $c(x, 2y)$

2. Suzie's remaining credit will also take into account charges for standard national SMS texts (t) and excess data charges (d).

Standard national SMS	$ 0.29
Excess data usage fee (per MB)	$ 2.00

a) Write the function rule for

$c(N, M, T, D) =$

b) Modify or redefine your Prime function and complete the table.

	Number of calls	Call minutes	SMS	Excess Data (Mb)	Remaining Credit ($)
$c(10,150,75,0)$					
$c(10,90,350,3)$					
	72	175	21	4	
$c(32,100,60,\quad)$					$107.12
	21		73	0	$43.53

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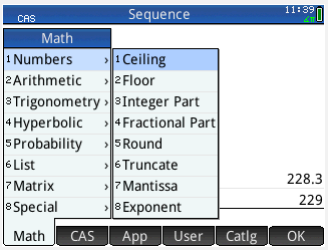
EXTENSION

FourMobile would want call minutes calculated automatically. It would be calculated using the integer part of a number function.

On Prime CEILING returns the smallest integer greater than or equal to the input. For example CEILING(228.3) returns 229.

In CAS mode:

press (Mem) tap Math **> Numbers > Ceiling**



3. Determine the value for each of the following function statements and compare with the table in Q1.

   a) CEILING(6.54)

   b) CEILING (18.25)

   c) CEILING (0.05)

   d) CEILING (0.42)

   e) CEILING (12+15/60)

   f) CEILING (2.00)

4. Define a function to calculate call minutes given the duration of a call as a decimal.

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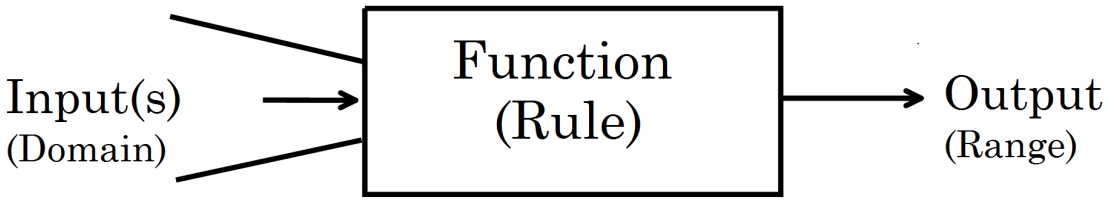
Learning Notes

Mathematical functions involve one or more inputs that generate one output or value. For example y-values of a function graph depend upon x.
In three dimensions a z-value is likely to be a function of x and y.

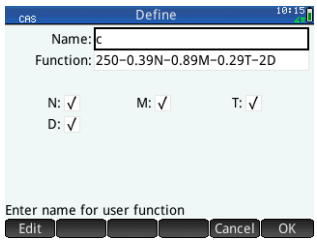
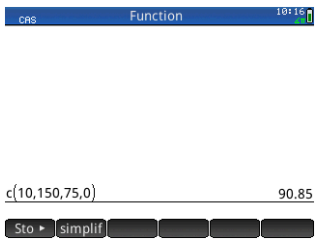


The Credit function in this investigation depends upon two factors: number of calls and call minutes. This assists in providing a realistic context to explore function notation and to appreciate that functions produce a single output.
Most of the functions you will study in this course are single variable functions. This topic includes linear, quadratic and cubic functions.

Functions in Prime:

Avoid single capital letters for function names as these are already set up as variables.

Q6

Define the function with 4 variables • Press CAS Settings to open CAS • Press Shift $xt0n$ Define D and press Enter ≈ to call the function the function c • Use the keyboard to enter 250–0.39N–0.89M–0.29T–2D for the expression. (Use Shift ALPHA alpha to enter variables as capital letters) and tap Enter ≈)	
Evaluate function • In CAS window enter the function name • enter the values given E.g. enter c(10,150,75,0) to find the credit after 10 calls, 150 call minutes, 75 SMS's and 0 Mb of extra data.	

Suzie's pre-paid account with *FourMobile* has $250 value. The table below shows how Suzie is charged for her calls.

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Activity 28 Moderating marks

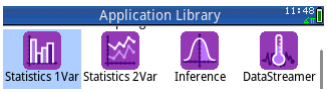
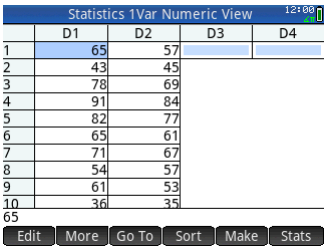
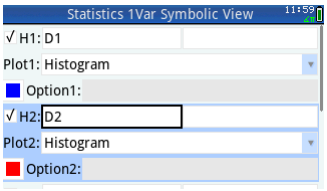
Aim: Compare data sets and adjust for comparability.

The table shows marks for a class of students. The first mark is determined by the teacher while the second mark for each student is their exam mark.

	Student																
Mark	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
School	65	43	78	91	82	65	71	54	61	36	88	69	58	65	79	45	62
Exam	57	45	69	84	77	61	67	57	53	35	75	65	52	62	71	41	55

How did the class' exam marks compare to the school mark? This activity will enable you to answer this type of question more precisely using mean or median for central tendency, and standard deviation or inter-quartile range for spread.

1. Enter the data into Statistics 2 Var app in Prime.

Enter the data • Select Statistics 1Var from the Apps Info menu	
Clear data and setup • Press Shift Esc Clear to clear data • Enter the student data in lists D1 and D2	
Calculate summary statistics • Press Symb Setup to enter symbolic mode • Press Shift Esc Clear to clear settings • Match settings to screen shot • Press Num Setup • Tap Stats to view summary statistics	

a) Complete the following table.

	Prime symbol/formula	School marks	Exam marks
Mean	$\overline{x}$		
Median	*Med*		
Standard deviation	σ_x		
Inter-quartile range	$IQR = Q_3 - Q_1$		

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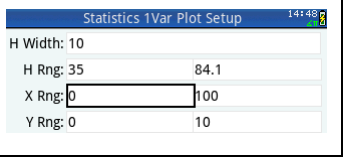
b) Use the statistics in part a) to argue that the class' school marks are generally better than their exam marks.

c) State which set of marks is more spread and justify your answer.

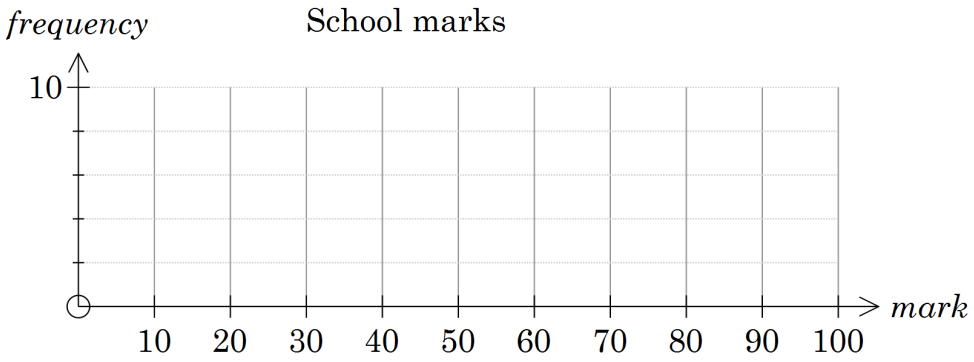
2. Compare histograms for school and exam marks:

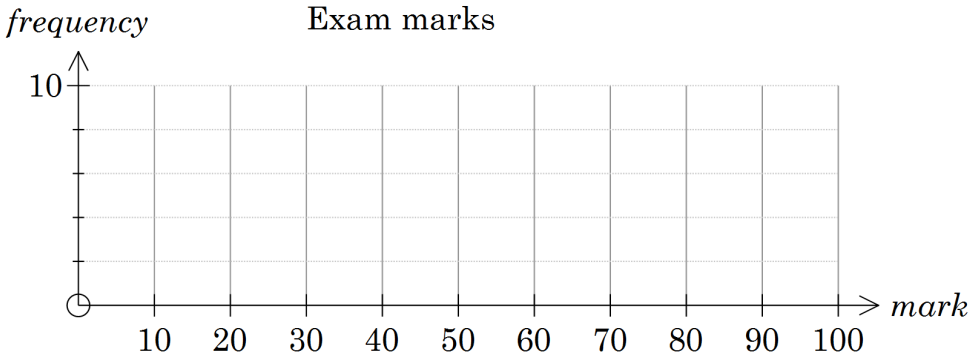
Construct histogram

- Press Shift Plot Setup to set up the graph
- Press Symb Setup to select list(s) to graph
- Press Plot Setup to graph



a) Draw the histograms generated on Prime.





b) Mark on your graphs the "middle" of each distribution.

c) What comparison between the two sets of marks do the histograms suggest?

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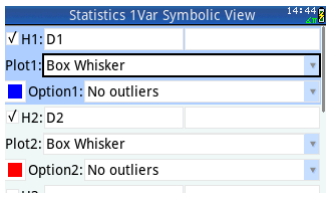
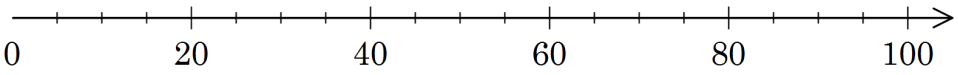
3.

a) Construct boxplots of the school and exam marks on your Prime and transcribe the plots to the number line below.

Construct boxplots • Press Symb Setup • Change plots to Box Whisker • Select both lists • Press Plot Setup	

Exam marks

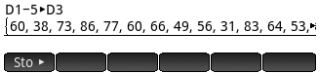
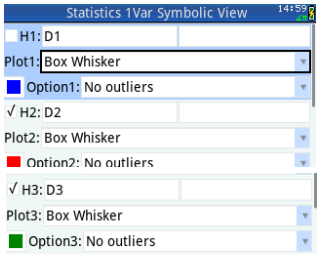
School marks



b) Which plot has the higher median?

In Year 12 school marks are often "moderated" by the exam. While the "moderation" process is complex the aim is to adjust each student's school mark so that the mean of the "adjusted" marks have the same mean and spread as the exam marks for the class.

4. Adjust school marks to have the same mean as the exam and store in list3.

Subtract 5 from each of the school marks and store in list3 • Press Settings • Enter D1–5 tap Sto ▸ and enter D3 this will take 5 from each element in list D1 and store as list D3	
Redraw boxplot • Select Statistics 1Var app • Press Symb Setup and set parameters for list D3 • Select lists D2 and D3 • Press Plot Setup	

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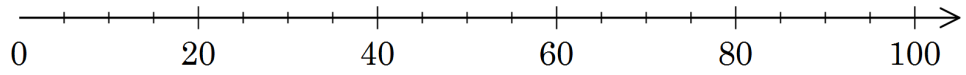
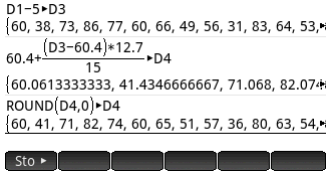
a) Redraw the boxplots. For list 1 and list 3. What has changed?

Exam marks

School marks - 5



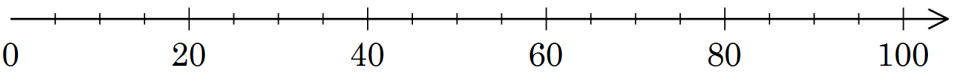
Adjust the spread (standard deviation) to match the exam marks and store in list D4.

Adjust the spread • Enter $60.4+(\text{D3}-60.4)\times\frac{12.7}{15}\rightarrow \text{D4}$ • This gives decimal answers • Enter ROUND(D4,0) D4 to round values to whole numbers (0 decimal places)	
Draw box plot • Set up and draw box plots of lists D3 and D4	

Redraw the boxplots for lists 1 and 4. How do they compare now?

Exam marks

Mod School marks



b) Enter the values from list4 into the moderated school mark row of the table.

	Student																
Mark	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
School	65	43	78	91	82	65	71	54	61	36	88	69	58	65	79	45	62
Exam	57	45	69	84	77	61	67	57	53	35	75	65	52	62	71	41	55
Moderated school mark																	

Which students did comparatively better in the exam than during the year? I.e. their exam mark was higher than their moderated school mark.

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5. In another school the marks have been summarised into class intervals.

Interval	Mid Interval	Frequency	
		School mark	Exam mark
31 - 35	33	2	0
36 - 40	38	1	0
41 - 45	43	5	4
46 - 50		17	12
51 - 55		35	28
56 - 60		48	48
61 - 65		39	45
66 - 70		24	28
71 - 75		9	13
76 - 80		3	3
81 - 85		1	1
86 - 90		0	1
91 - 95		1	0

a) Complete the column Mid Interval in the above table.

b) Calculate summary statistics and complete the following table.

	School marks	Exam marks
Mean		
Standard deviation		

c) Did the students generally get better school or exam marks? Justify your answer.

d) Which set of marks is more spread?
Justify your answer.

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Solutions

Activity 1 Basic calculations quiz

1. 50.22
2. 10
3. 4.123
4. 5
5. –0.688
6. 7
7. 15.288
8. 8.988
9. 0.439
10. 4.632

$16.200*3.100$	50.220
$7.000+2.000*3.000^{2.000}-15.000$	10.000
$\sqrt{17.000}$	4.123
$\sqrt{9.000+16.000}$	5.000
$\frac{963.100}{171.600}-6.300$	-0.688
$\sqrt{9.000}+\sqrt{16.000}$	7.000
$3.910^{2.000}$	15.288
Ans−6.300	8.988
$\frac{3.910}{8.900}$	0.439
$\frac{3.910+1.077}{1.077}$	4.632
50.220+10.000+4.123+7.000+-0.688+5.000+15.2▸	105.003

In standard mode

3.91▸A	3.91
1.0765▸B	1.0765
6.3▸C	6.3
8.9▸D	8.9
16.2*3.1	50.22
$7+2*3^{2}-15$	10
$\sqrt{17}$	4.12310562562
$\sqrt{9}+\sqrt{16}$	7
$\frac{963.1}{171}-6.3$	-0.66783625731
$\sqrt{9+16}$	5
A^{2}	15.2881
Ans−C	8.9881
$\frac{A}{D}$	0.439325842697
$\frac{A+B}{B}$	4.63214119833
50.22+10+4.12310562562+7+-0.66783625731+5▸	105.022936409

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Activity 2 Percentages quiz

1. $28.38
2. 12.41%
3. $17.89
4. $16.80
5. $112.50
6. 67.64
7. $549.86
8. $50
9. $6750
10. $8700

$12{,}340.000 * 0.002$	28.382
$\frac{521{,}000.000 - 463{,}500.000}{463{,}500.000}$	0.124
$\frac{521{,}000.000 - 463{,}500.000}{463{,}500.000} * 100.000$	12.406
$17.340 * 1.032$	17.895
$\frac{17.340}{1.032}$	16.802
$120.000 * 0.750 * 1.250$	112.500
$64.300 * 1.052$	67.644
$796.900 * (1.000 - 0.310)$	549.861
$550.000 - \frac{550.000}{1.100}$	50.000
$450{,}000.000 * 0.015$	6,750.000
$3{,}500.000 + 0.008 * 650{,}000.000$	8,700.000

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Activity 3 Best buys

1.

a) 0.71

b) 0.67

c) Round 5

$\frac{5}{7}$	0.71
$\frac{6}{9}$	0.67
$\frac{224.000}{334}$	670.66
$\frac{245.000}{361}$	678.67
$\frac{240.000}{352}$	681.82

2.

a) $670.66

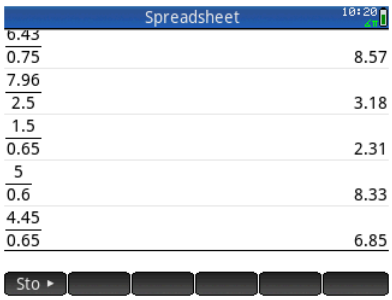
b) $671.23

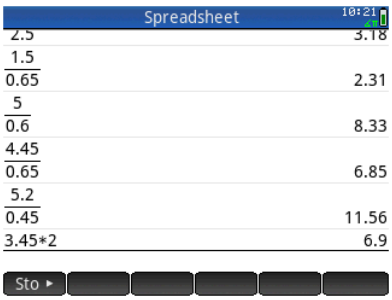
c) $681.82

d) Block A is cheapest

3.

Item	*Quantity*	*Price*	*Price per unit*
Cooking Oil	750 ml	$6.43	$8.57 / L
Fruit Juice	2.5 L	$7.96	$3.18 / L
Chocolate bar	65 g	$1.50	$2.31 / 100g
Flavoured Milk	600 ml	$2.50	$4.17 / L
Bread	650 g	$4.45	$6.85 / kg
Peanut Butter	450 g	$5.20	$1.16 / 100g
Flour	2 kg	$6.90	$3.45 / kg





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Activity 4 Maddy's boots

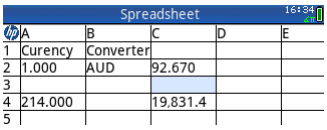
1.

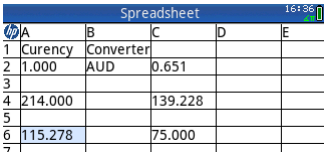
a) Cell C4 changes value. It displays what the number of $A is equivalent to.

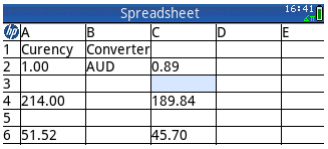
b) Cell A6 changes value. It displays the Australian dollar equivalent.

2.

A$	Other currency
A$214	19831.38 Yen
A$115.28	75 Euros
A$23.50	22138.41 won
A$1093	592.62 Pound sterling
A$10.68	200 000 Vietnamese dong
A$75	815025 rupiah



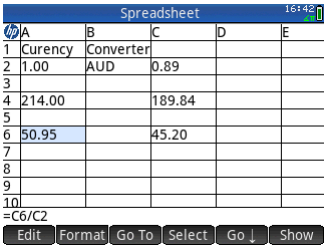


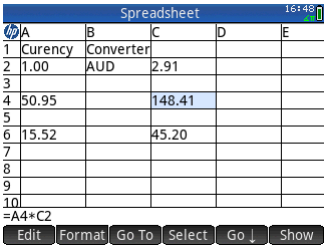


3. $51.52

4. Answers will vary depending upon current rates.

5. $148.41





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Activity 5 Maxine's car

1.

a) 15.4 litres

b) $25.41

2.

a) =B2×B4/100

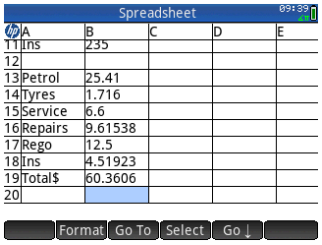
b) Same

3. $16.70

4.

a) $60.36

b) She cannot afford it as her budget allows only $50 per week,



5. Answers will vary.

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Activity 6 Algebra quiz

1. 15.288
2. 8.988
3. 0.439
4. 4.632
5. 5.014
6. 1.837
7. 2.729
8. 4.987
9. 0.150
10. 3.956

A^2	15.2881
Ans−C	8.9881
$\frac{A}{D}$	0.439325842697
$\frac{A+B}{B}$	4.63214119833
10−(A+B)▸E	5.0135
$\frac{\sqrt{A}}{B}$▸F	1.83685275739
$\frac{E}{F}$▸G	2.72939677926
Solve.SOLVE(A=H−B,H)	4.9865
Solve.SOLVE(E=A+K∗(B+C),K)	0.149596692198
Solve.SOLVE(G+K=L−B,L)	3.95549347146

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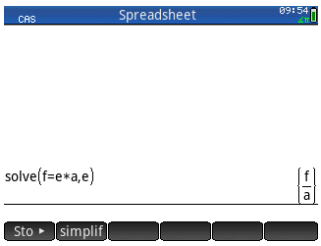
Activity 7 Currency trade

1. US$270

2. A1$ is 2.81 ringgit

3.



a) $e = \frac{f}{a}$

b) $a = \frac{f}{e}$

4.

a) C9: =C7 × C8

b) C14: =C12/C13

c) C19: =C17/C18

5.

a) $r = \frac{C}{2\pi}$

b) $w = \frac{P}{2} - l$

c) $l = \sqrt{A}$

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Activity 8 Taxi charges

1. $2.568 or $2.57

2.

Day/night	hire charge H	per km rate r	kms travelled d	minutes waiting m	maximum fare F
day	$3.50	$2.14	8.6	2	$23.75
night	$6.00	$2.568	12	0	$36.82
day	$3.50	$2.14	14	5	$38.07
night	$6.00	$2.568	4.5	1	$18.48
day	$3.50	$2.14	13	12	$42.37
night	$6.00	$2.568	7.2	17	$40.15

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Activity 9 Body mass index

1. Answers will vary

2. Answers will vary. An example is shown.

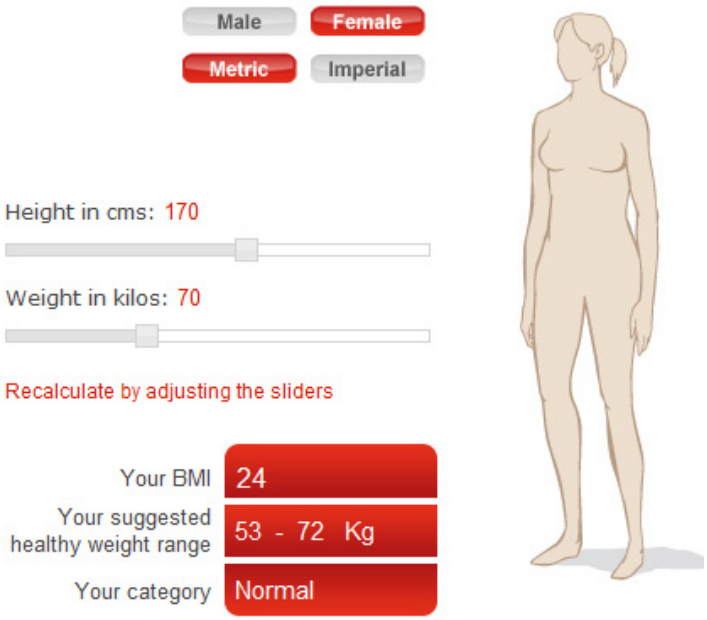
3.

Spreadsheet 06:39

	A	B	C	D	E
1	BMI	Converter			
2			Height inc		0.01
3			Weight inc		2
4		Weight			
5	Height	63	65	67	69
6	1.7	21.79931	22.49135	23.18339	23.875
7	1.71	21.54509	22.22906	22.91303	23.597
8	1.72	21.29529	21.97134	22.64738	23.323
9	1.73	21.04982	21.71807	22.38631	23.054
10	1.74	20.80856	21.46915	22.12974	22.790

=(B$5)/($A6^2)

Edit | Format | Go To | Select | Go ↓ | Show



4.

Height	Weight	BMI
1.78 m	75 kg	24 (23.7)
164 cm	62 kg	23
1.66 m	51	18.5
1.84	85kg	21

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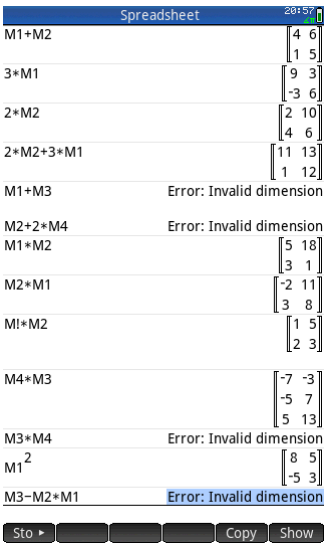
Activity 10 Matrix arithmetic



1. $\begin{bmatrix} 4 & 6 \\ 1 & 5 \end{bmatrix}$

2. $\begin{bmatrix} 9 & 3 \\ -3 & 6 \end{bmatrix}$

3. $\begin{bmatrix} 2 & 10 \\ 4 & 6 \end{bmatrix}$

4. $\begin{bmatrix} 11 & 13 \\ 1 & 12 \end{bmatrix}$

5. Not possible *invalid dimension*
The matrices have different sizes,
i.e. C has 3 rows and A has 2 rows.

6. Not possible *invalid dimension*
The matrices have different sizes.

7. $\begin{bmatrix} 5 & 18 \\ 3 & 1 \end{bmatrix}$

8. $\begin{bmatrix} -2 & 11 \\ 3 & 8 \end{bmatrix}$

9. B×C is not possible.

10. $\begin{bmatrix} -7 & -3 \\ -5 & 7 \\ 5 & 13 \end{bmatrix}$

11. Not possible *invalid dimension*. The number of columns in the first matrix is not the same as the number of rows in the second matrix.

12. $\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

13. Not possible *invalid dimension* B×A is a 2×2 matrix whereas C is 3×2.

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Activity 11 Premiership table

1. a)

	Won	Drawn	Lost
Chelsea	7	3	2
Tottenham Hotspur	6	2	4
Arsenal	10	1	2
West Ham United	3	4	6
Crystal Palace	2	1	10
Fulham	3	1	9

   b) 6

   c) 3

2.

   a) 5

   b) 0

   c) 11

3.

   a) $\begin{pmatrix} 7 & 3 & 2 \\ 6 & 2 & 4 \\ 10 & 1 & 2 \\ 3 & 4 & 6 \\ 2 & 1 & 10 \\ 3 & 1 & 9 \end{pmatrix} + \begin{pmatrix} 5 & 2 & 3 \\ 7 & 0 & 3 \\ 4 & 5 & 2 \\ 4 & 3 & 2 \\ 5 & 1 & 3 \\ 2 & 2 & 6 \end{pmatrix} = \begin{pmatrix} 12 & 5 & 5 \\ 13 & 2 & 7 \\ 14 & 6 & 4 \\ 7 & 7 & 8 \\ 7 & 2 & 13 \\ 5 & 3 & 15 \end{pmatrix}$

   b) The results of the six teams over both periods.

4.

   a) 9

   b) 4

5. a) $\begin{bmatrix} 9 & 6 & 6 \\ 12 & 6 & 4 \end{bmatrix}$

   b) The results between Christmas and the end of the season.

6. Arsenal has 37, Tottenham has 27

7.

   a) $\begin{bmatrix} 37 \\ 27 \end{bmatrix}$ b) $\begin{bmatrix} 70 \\ 69 \end{bmatrix}$

8. The number of points each team has at that point in the season.

9. a) Add elements in the same row and column together. It only makes sense if each matrix has the same number of rows and columns.

   b) Subtract elements in the same row and column.

   c) Go across a row in the first matrix and down the column in the second. Multiply each pair together and then add.

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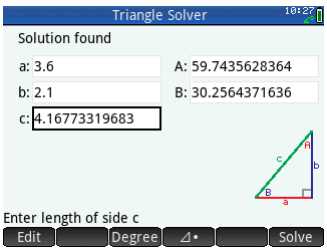
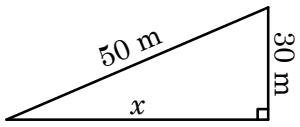
Activity 12 Pythagoras

1. 4.17 m



2. a)

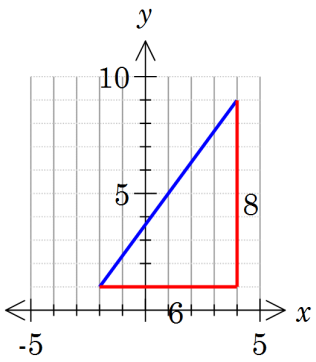


$x^2 + 30^2 = 50^2$

$x = 40$

The kite would be 40 m from Luc.

b)

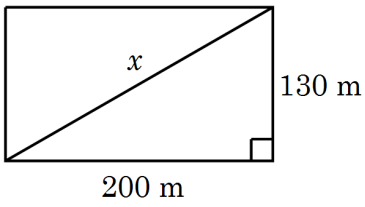


$6^2 + 8^2 = x^2$

$x = 10$

The points are 10 units apart

c)

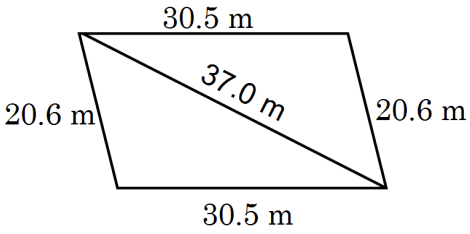


$x^2 = 200^2 + 130^2$

$x = 238.5$

The long route is 3.3 units and is 0.9 units longer (1 decimal place)

d)



If it is a right angle then Pythagoras' theorem will work

Does $20.6^2 + 30.5^2 = 37.0^2$?

No. It is close and the angle is 90.7°, off by less than 1°.

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Activity 13 Measurement formulas

1.

a) 378 cm^3

b) 295 cm^2

2. 4.16 m^3

3. 134+261.8+452.4+718.3+2827.4 = 4390 cm^3 (nearest 10 cm^3)

4.

a)

(i) 254 cm^2

(ii) 382 cm^3

b)

(i) 1018 cm^2

(ii) 3054 cm^3

c) ¼ as it has four times the area to cover

d) The ratio of the volumes is 8:1

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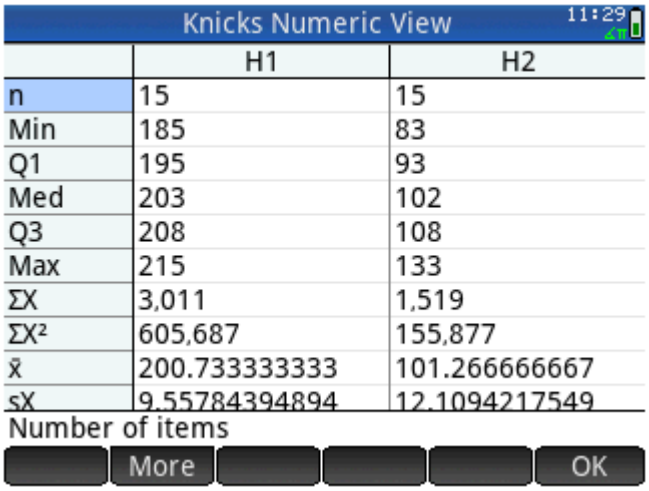
Activity 14 Knicks'tistics

1.

	Height	Weight
Mean	201 (nearest cm)	101 (nearest kg)
Median	203	102
Maximum	215	133
Range	215 – 185 = 30	133 – 83 = 50
Inter quartile range	208-195 = 13	108 – 93 = 15



2. 3011 cm or 30.11 m. The total of all the scores is ΣX.

3. Total weight is 1519 kg or 1.5 tonnes. This is more than 1 tonne.

4. 15 players. n=15.

5. Answers will vary.

   a) These results are for a height of 175 cm and weight 75 kg.

	Height	Weight
Mean	199 (nearest cm)	100 (nearest kg)
Median	201.5	100.5
Maximum	215	133
Range	215 – 175 = 40	133 – 75 = 58
Inter quartile range	208 – 191 = 17	105 – 93 = 12

   b) The range is most affected in the above example.

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Activity 15 Knicks'tistics II

1.

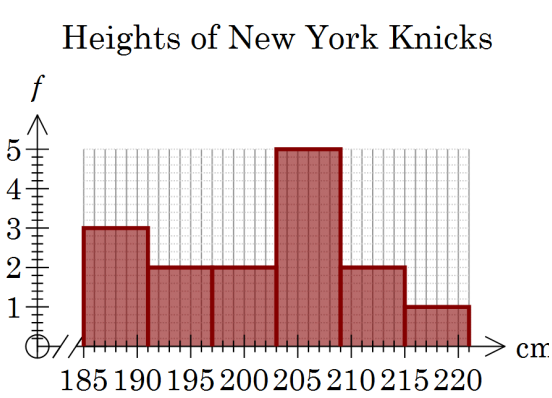
Heights

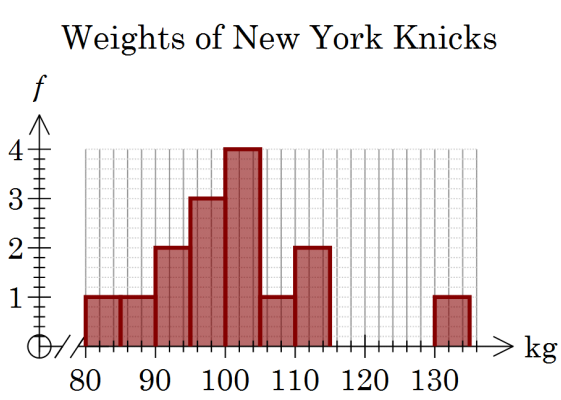
Interval		Frequency
Start	End	
185	190	3
191	196	2
197	202	2
203	208	5
209	214	2
215	220	1

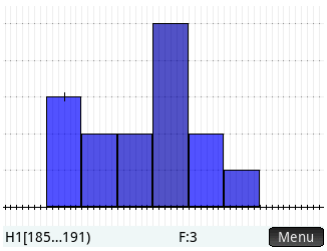
Weight

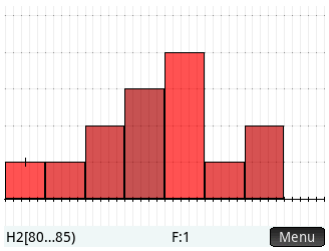
Interval		Frequency
Start	End	
80	84	1
85	89	1
90	94	2
95	99	3
100	104	4
105	109	1
110	114	2
> 115	-	1

2.









3.

a) One player is very heavy.

b) Looking at the distribution the middle is approximately in the middle of the bars.

c) Height 185 – 210 cm, weight : 90 – 115 kg

d) The data is clumped apart from one player who is very heavy. The histograms suggest the distribution is similar to a bell shaped curve.

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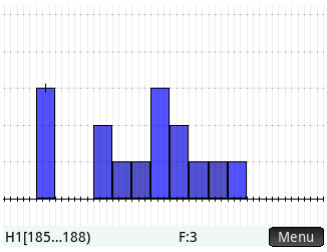
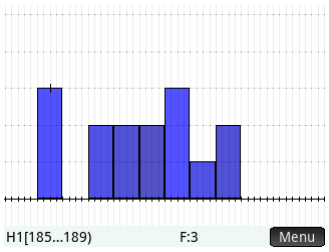
4.

a) Heights starting at 185 cm with interval width of

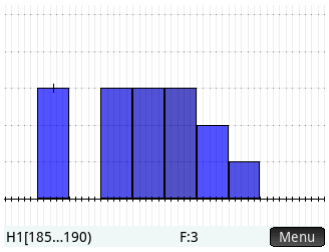
- 3



- 4

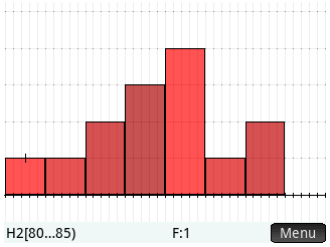


- 5

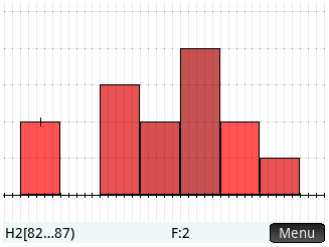


b) Weights using 5 for the interval width and starting at

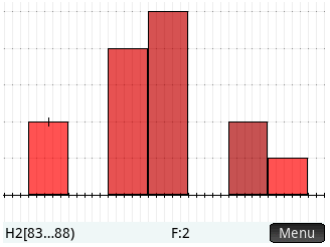
- 80



- 82

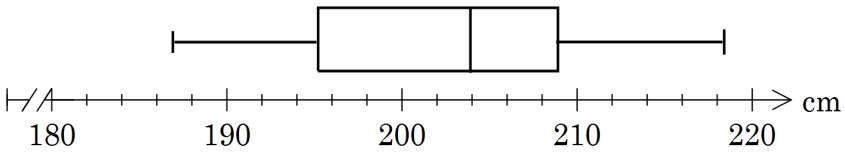


- 83



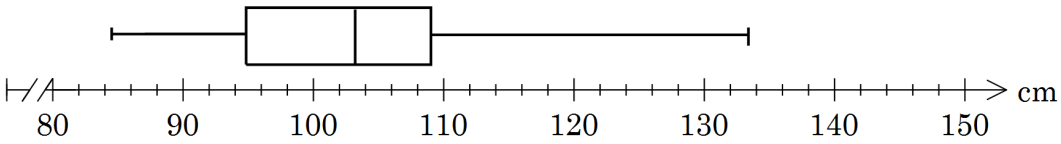
5. a)

Box Plot: Heights of NY Knicks roster



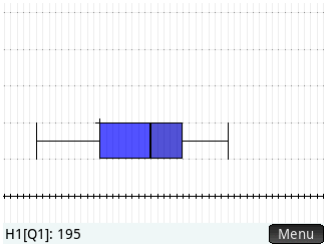
b)

Box Plot: Weights of NY Knicks roster

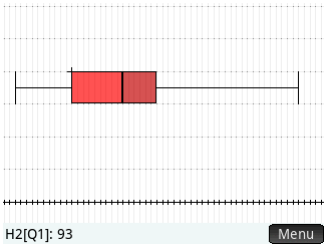


Screen shots from Prime

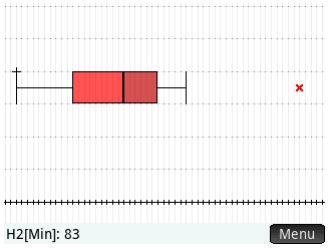
Height



Weight



Weight - show outliers



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Activity 16 Rowers v Knicks

1.

a)

	Heights in cm	
	Basketballers	Rowers
Mean	200.7	189.4
Median	203	192
Maximum	215	198
Range	30	32
Inter-quartile range	13	8
Standard deviation	9.2	8.4

b) Mean: basketballers are taller.
Median: basketballers are taller.
Maximum: the tallest person is a basketballer
Range: similar difference between smallest and tallest for each group.
IQR: basketballers are greater suggesting a greater variation or spread of heights
Standard deviation: basketballers greater suggesting greater spread of heights.

c) The basketballers are generally taller, and more varied (or spread) in their heights.

2.

a)

	Weights in cm	
	Basketballers	Rowers
Mean	101.3	86.4
Median	102	90
Maximum	133	94
Range	50	39
Inter quartile range	15	7
Standard deviation	11.7	11.5

b) Mean: basketballers are heavier.
Median: basketballers are heavier.
Maximum: the heaviest person is a basketballer

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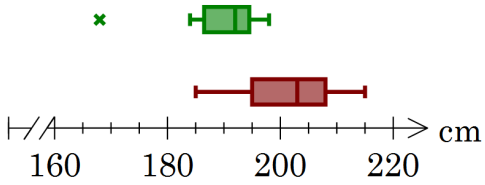
Range: a greater difference between lightest and heaviest for the basketballers.
IQR: basketballers are greater suggesting a greater variation or spread of weights
Standard deviation: basketballers greater suggesting greater spread of weights.

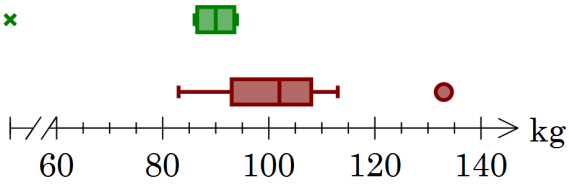
c) The basketballers are heavier with a greater spread of weights.

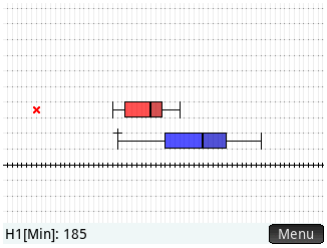
3.

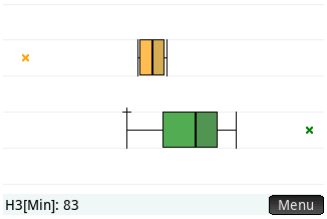
Height: rowers & basketballers



Weight boxplot: rowers & basketballers







4. The side by side boxplots show the differences effectively as a picture. The summary statistics are often more difficult to use to justify a comparison.

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Activity 17 Reaction times

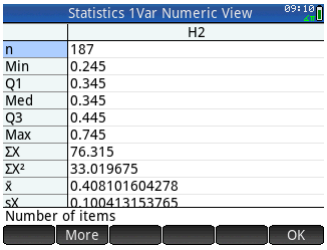
1. 0.4

2. 0.1

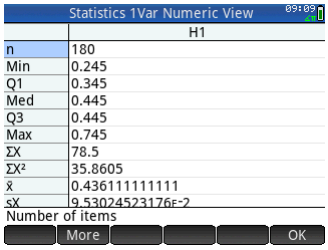
3.

Statistic		Dominant hand	Non-dominant hand
a)	Mean	0.408	0.436
b)	Minimum	0.245	0.245
c)	Lower Quartile	0.345	0.345
d)	Median	0.345	0.445
e)	Upper Quartile	0.445	0.445
f)	Maximum	0.745	0.745
g)	Range	0.5	0.5
h)	Inter-quartile range	0.1	0.1
i)	Standard deviation	0.1	0.095

Dominant hand

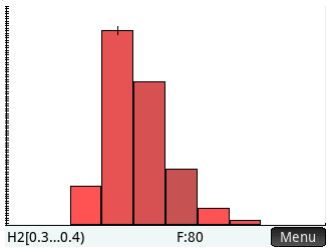


Non-dominant hand

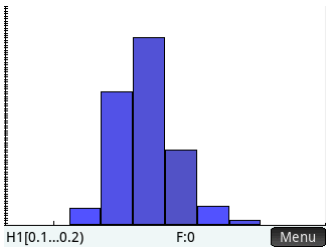


4.

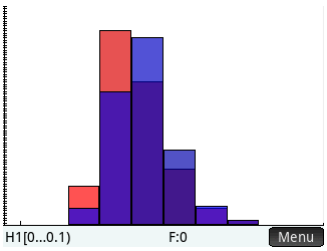
a) Dominant hand



Non dominant hand

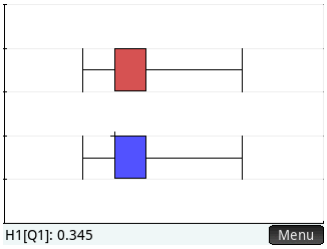


Both hands



b) The histogram shows a shift to the right with the non-dominant hand. That is the dominant hand is faster.

5.



The boxplots suggest no difference in reaction times.

6. The histograms using the same scale on the horizontal axis show reaction times with the dominant hand to be generally faster. The box plot shows no difference.

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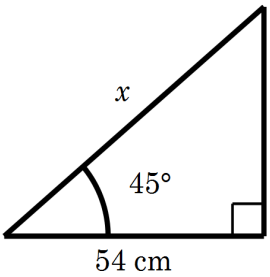
Activity 18 Right-angled triangles

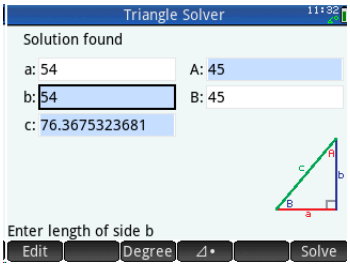
1.



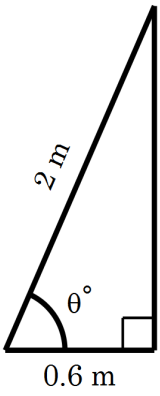
$$\sin 45° = \frac{54}{x}$$

$$x = 76.368 \text{ cm}$$

The cut is 76.4 cm long.



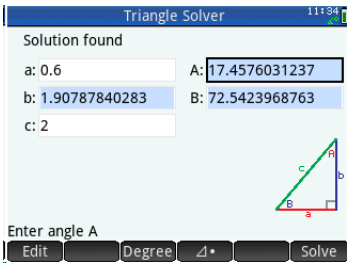
2.



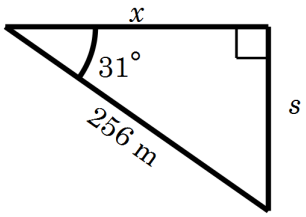
$$\sin\theta° = \frac{0.6}{2}$$

$$\theta = 72.54$$

The ladder is making an angle of 73° to the ground. (nearest degree)



3.

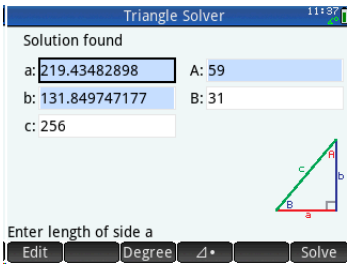


$$\sin 30° = \frac{s}{256}$$

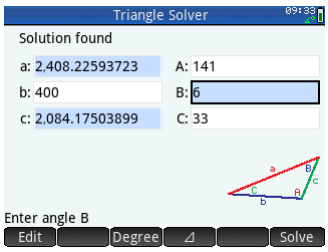
$$\cos 31° = \frac{x}{256}$$

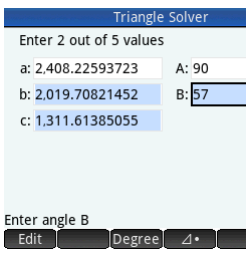
$$s = 131.85, x = 219.435$$

X is 131 m South and 219 m East of A (to the nearest metre)



4. The mountain summit is 1300 m higher





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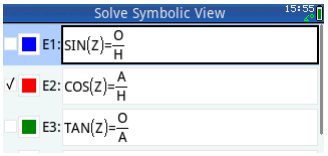
Activity 19 Right-angled triangles II

1.

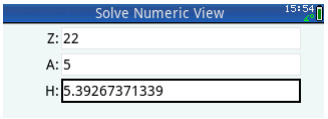
a)



$$\cos\theta = \frac{A}{H}$$

$$\cos 22° = \frac{5}{x}$$

$$x = \frac{5}{\cos 22} = 5.39$$

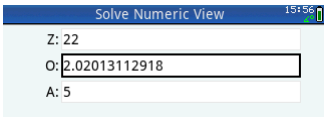


The distance from the apex to the outside of the wall is 5.39 m,. i.e. a 6m length timber is long enough.

b)

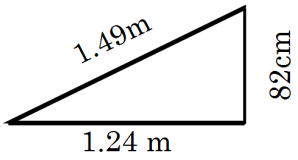
$$\tan 22° = \frac{h}{5}$$

$$h = 2.02$$



The apex is 2.02 m above the wall.

2. a)



b)

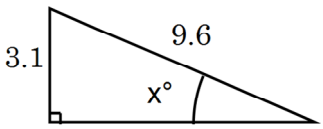
$$\tan\theta = \frac{O}{A}$$

$$\tan\theta = \frac{0.82}{1.24}$$

$$\theta = 33.5°$$

The smallest angle is 33.5°

3. a)



b)

$$\sin x° = \frac{3.1}{9.6}$$

$$x = 18.8°\ \text{(1 dec. place)}$$

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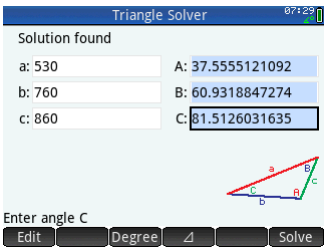
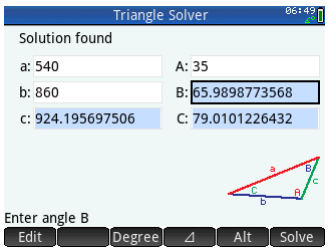
Activity 20 Window dressing

1.





a) 81.5°

b) 60.9°

c) 71.9 cm

d) 95.4 cm

e) 4089 cm²

f) $80.35

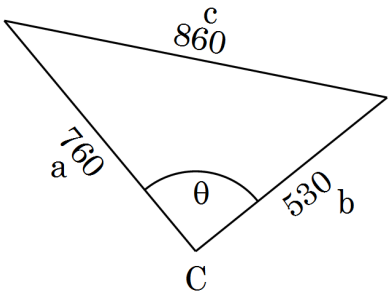
2.

a) Labelling shown on diagram.
Substituting into

$c^2 = a^2 + b^2 - 2ab\cos C$

$86^2 = 76^2 + 53^2 - 2\times 76\times 53\cos\theta$

b) $\theta = 81.5°$



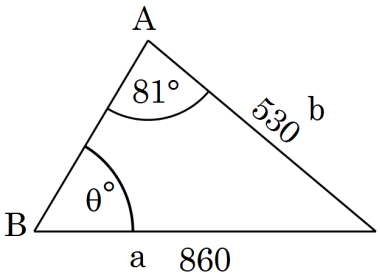
3.

a) Label as shown and substitute gives

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 81°}{860} = \frac{\sin\theta}{530}$$

b) 37.5 °

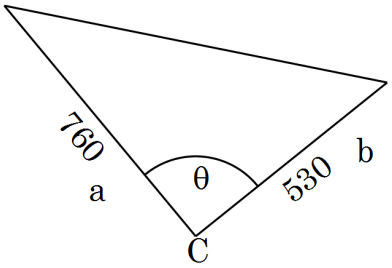


4.

a) $Area = \frac{1}{2}ab\sin C$

$1995 = \frac{1}{2}76\times 53\times\sin\theta$.

b) $\theta = 82.1°$



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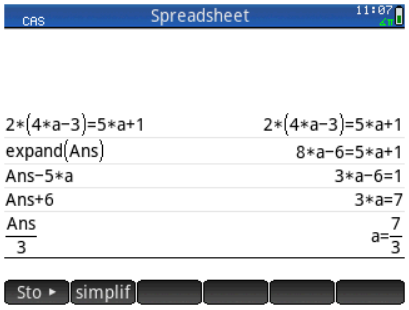
Activity 21 Solving equations

1.

$$25 - 2x = 17$$
$$-2x = 8$$
$$x = -4$$

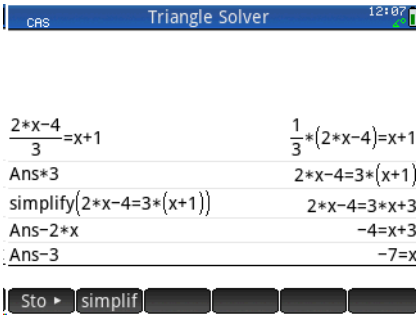
2.

$$2(4a - 3) = 5a + 1$$
$$8a - 6 = 5a + 1$$
$$3a - 6 = 1$$
$$3a = -7$$
$$a = \frac{-7}{3}$$



3.

$$\frac{2y - 4}{3} = y + 1$$
$$2y - 4 = 3(y + 1)$$
$$2y - 4 = 3y + 3$$
$$-4 = y + 3$$
$$y = -7$$



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Activity 22 Features of straight lines

1.

Equation	Slope (sign)	Slope	y-intercept	x-intercept
$y = 3x - 3$	+	3	-3	1
$y = 7 - 0.3x$	–	–0.3	7	23.3
$x + y = 5$	–	–1	5	5
$2y - 3x + 6 = 0$	+	1.5	–3	2
$x = 4y + 3$	+	0.25	–0.75	3
$y = 3.2$	0	0	3.2	none
$y = 2.2x + 5$	+	2.2	5	–2.27
$y = 2x + 10$	+	2	10	–5

2.

Points	Slope	Equation	y-intercept
(-2, -1) and (3, 2)	0.6	$y = 0.6x + 0.2$	0.2
(1, 3) and (6, 8)	1	$y = x + 2$	2
(3.2, 1.8) and (4, -0.6)	–3	$y = -3x + 11.4$	11.4
(2, 6) and (2, -5)	undefined	$x = 2$	none
(10, 0) and (0, 5)	–0.5	$y = -0.5x + 5$	5

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Activity 23 Deluxe Taxi fares

1.

a) The trip will cost 2.50 + 8 times 4, i.e. $34.50

b) $52.50

c) 12 km (nearest km)

2. As for Q1

3.

a)

(i) $27.70

(ii) $132.10

b)

(i) 2km (nearest km)

(ii) 19 km (nearest km)

4.

a) $33.20

b) 4 km (nearest km)

c) 25 minutes (nearest min)

5.

a) $C = 4.3 + 4.71d + .6m$

b)

(i) $39.95 (for cash)

(ii) 3 km (nearest km)

(iii) 9 minutes (nearest min)

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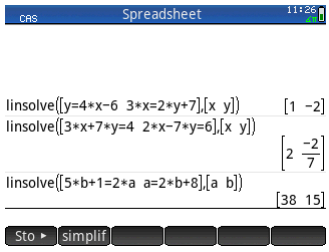
Activity 24 Simultaneous equations

1.

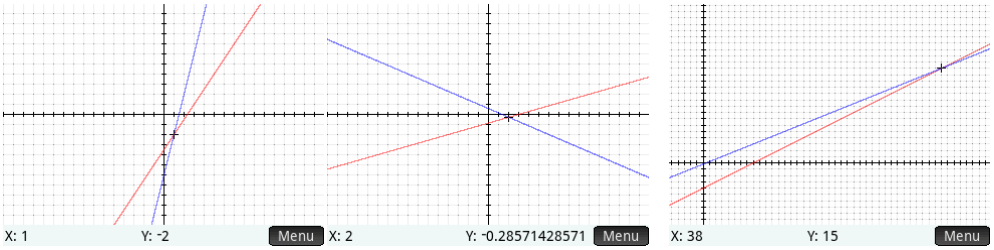
a) $x = 1, y = -2$

b) $x = 2, y = \frac{-2}{7}$

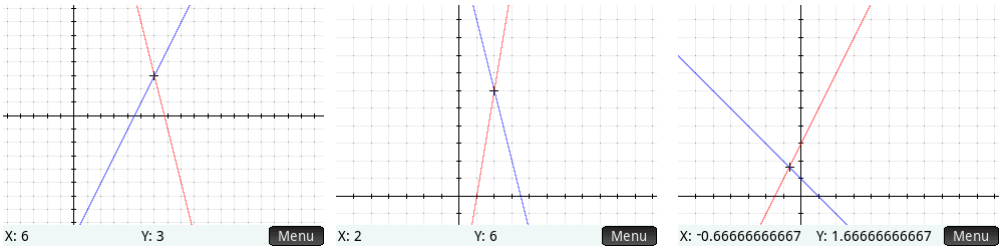
c) $a = 38, b = 15$



2. Same as Q1.



3. a) $x = 6, y = 3$ b) $x = 2, y = 6$ c) $x = -0.67, y = 1.67$

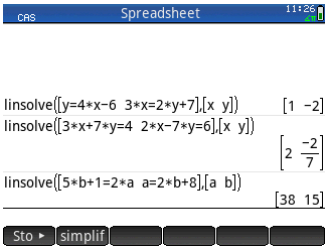


4.

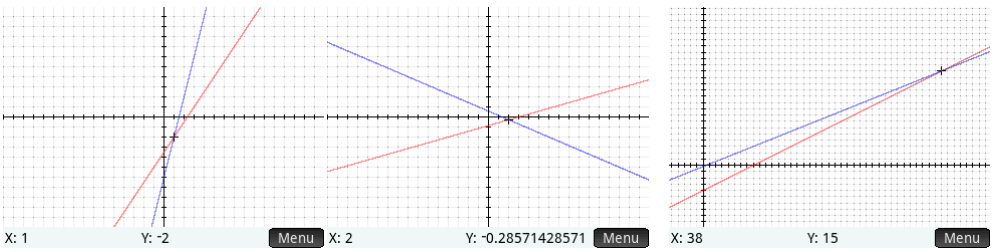
a) $x = 1, y = -2$

b) $x = 2, y = \frac{-2}{7}$

c) $a = 38, b = 15$



5. Same as Q1.



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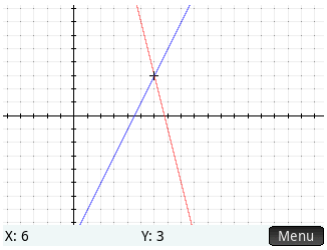
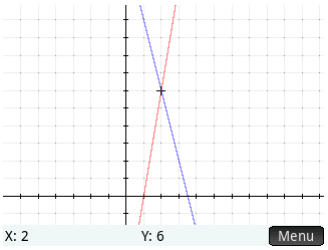
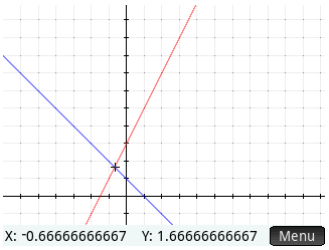
6. a) $x = 6, y = 3$ b) $x = 2, y = 6$ c) $x = -0.67, y = 1.67$







7.

a) $y = 4x - 6 \quad \ldots \quad 1$

$x = 2y + 5 \quad \ldots \quad 2$

sub 2 into 1

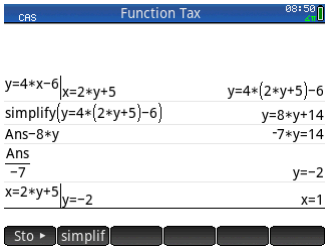
$y = 4(2y + 5) - 6$

$y = 8y + 14$

$-7y = 14$

$y = -2$

$x = 2(-2) + 5 = 1$



b) $x = y - 4 \quad \ldots \quad 1$

$3x - 2y = -8 \quad \ldots \quad 2$

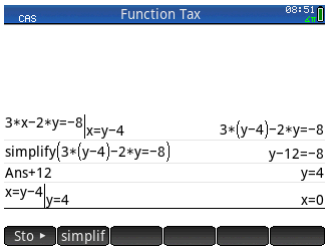
sub 1 into 2

$3(y - 4) - 2y = -8$

$y - 12 = -8$

$y = 4$

$x = 3(4 - 4) = 0$



c) $x = 2y + 1 \quad \ldots \quad 1$

$2y = 5x - 4 \quad \ldots \quad 2$

sub 1 into 2

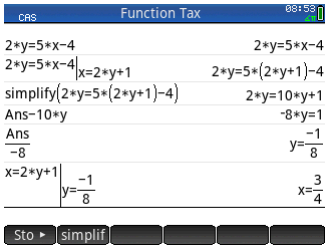
$2y = 5(2y + 1) - 4$

$2y = 10y + 1$

$-8y = 1$

$y = -\frac{1}{8}$

$x = 2\left(-\frac{1}{8}\right) + 1 = \frac{3}{4}$



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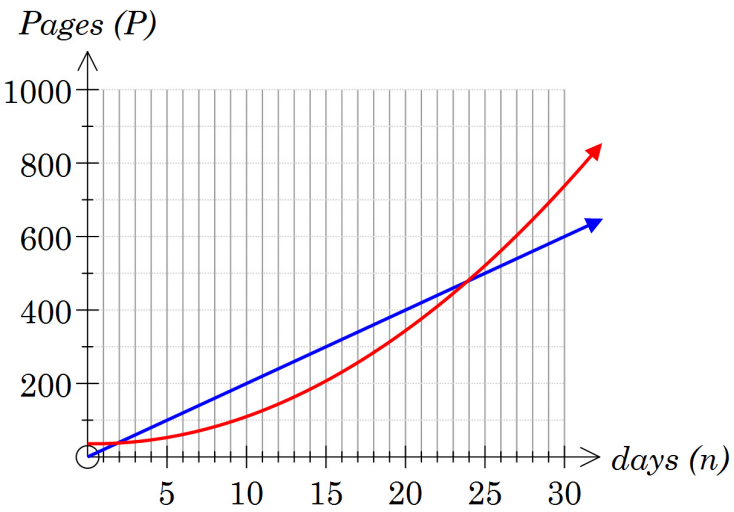
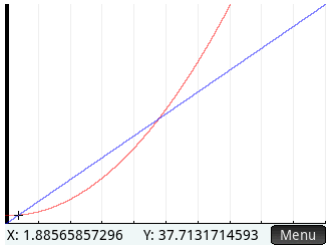
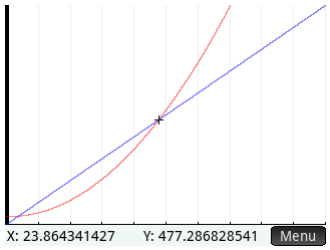
Activity 25 Book club

1. $P=20n$

2. & 3.

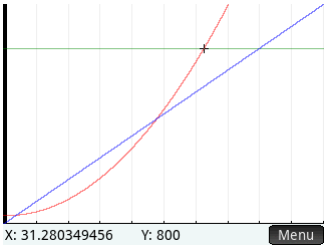




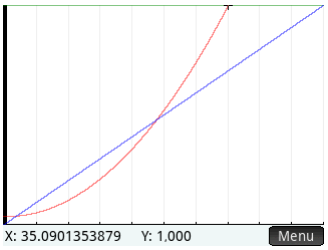


4. After 2 days and 24 days.

5. Jen takes 40 days. Ahjoy finishes on the 32nd day. So Ahjoy starts 8 days later.



6. Jen takes 50 days. Ahjoy finishes on the 36th day. So Ahjoy starts 14 days later.



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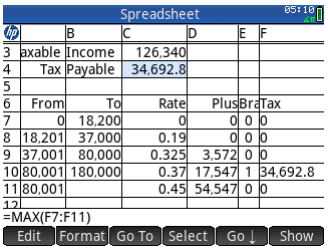
Activity 26 Income tax

1.

a) $0
b) $0.19
c) $21 912.26
d) $2680.90

2.

Name		Income	Tax
a)	Alessia	$26 065	$1494.35
b)	Bruce	$126 340	$34692.80
c)	Christine	$76 922	$16546.65
d)	Dylan	$16 980	0
e)	Edna	$40 694	$4772.55
f)	Fletcher	$234 560	$79099



3.

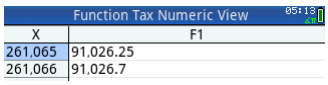
a) The amount in excess of $37000 is ($x$-37000).
The tax on this is 32.5 cents in the $ or 0.325(x-37000).
The total tax is $3572 plus 0.325($x$-37000)

b) i) $0.19(x-18200)$ ii) $17547+0.37(x-80\,000)$

c)

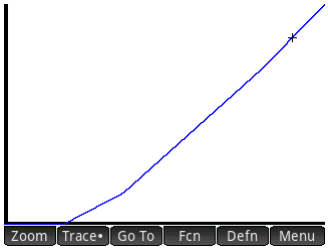
Name		Income	Tax payable
(i)	Gillian	$261 065	$91 026.25
(ii)	Hadi	$18 000	$0
(iii)	Ita	$19 100	$171
(iv)	Jack	$20 000	$342
(v)	Kate	$87 694	$20393.78
(vi)	Lam	$111 560	$29224.20

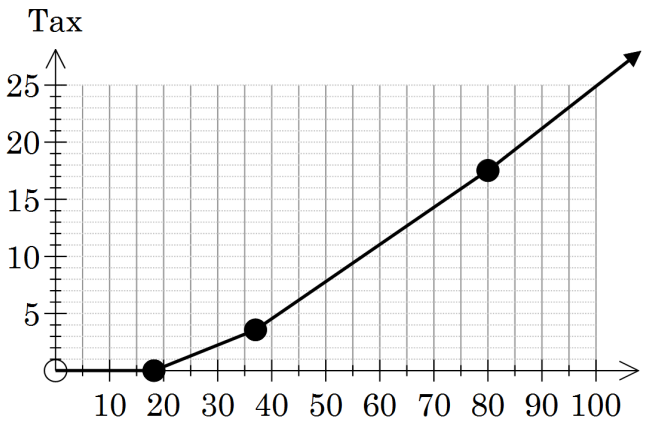
E.g.



4.

a) The graph is made up of straight lines (pieces)





b) the slope changes at 18 200, 37 000 and 80 000
This is because the rate changes at these amounts.

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Activity 27 Phone costs

1. a) 9
   b) 85
   c) $79.16
   d) $170.84
2.

	calls	minutes	Credit remaining
$c(10,250)$	10	250	$23.60
$c(50,150)$	50	150	$97
$c(72,175)$	72	175	$66.17
$c(32,220)$	32	220	$41.72
$c(40,200)$	40	200	$56.40

3. 70
4. a) $8.32
   b) $45.26
   c) 202
5. a) $-0.89m + 246.1$
   b) $-0.89\,\text{mins} + 246.1$
   c) $-0.39x - 0.89y + 250$
   d) $-1.78m + 246.1$
   e) $-0.39x - 1.78y + 250$
6. a) $c(n,m,t,d) = 250 - 0.39n - 0.89m - 0.29t - 2d$
   b)

	calls	minutes	SMS	Data Mb)	Credit ($)
$c(10,150,75,0)$	10	150	75	0	$90.85
$c(10,90,350,3)$	10	90	350	3	$58.50
$c(72,175,21,4)$	72	175	21	4	$52.08
$c(32,100,60,12)$	32	100	60	12	$107.12
$c(21,199,73,0)$	21	199	73	0	$43.53

7. a) 7
   b) 19
   c) 1
   d) 1
   e) 12
   f) 2
8. The function $c(n,m) = 250 - 0.39n - 0.89 \times \text{CEILING}(m)$ would enable m to be entered as a decimal rather than being rounded up first.

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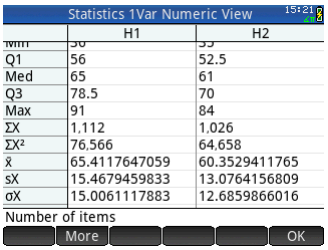
Activity 28 Moderating marks

1.

a)

	School mark	Exam mark
Mean	65.4	60.4
Median	65	61
Standard deviation	15	12.7
Inter-quartile range	78.5 – 56 = 22.5	70- 52.5 = 17.5

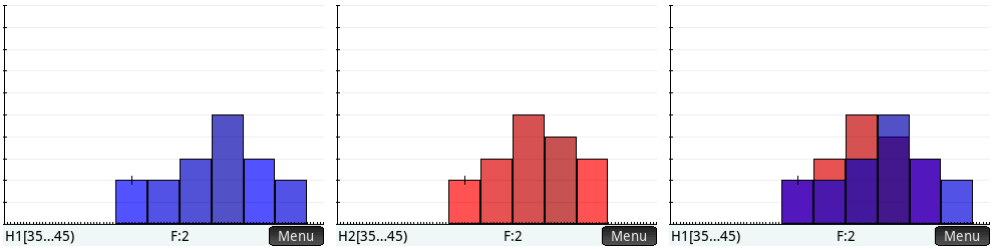


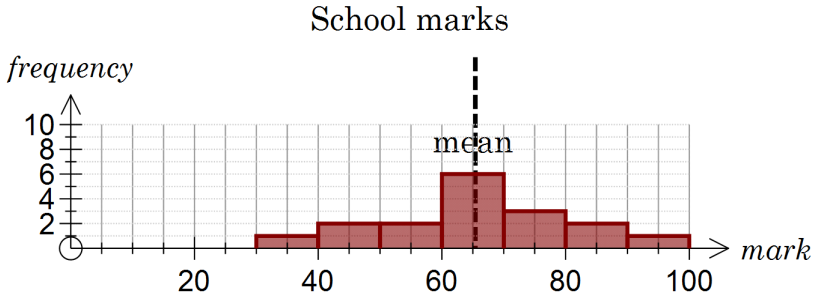
b) Both the mean and median for the exam are lower suggesting the class did less well on the exam.

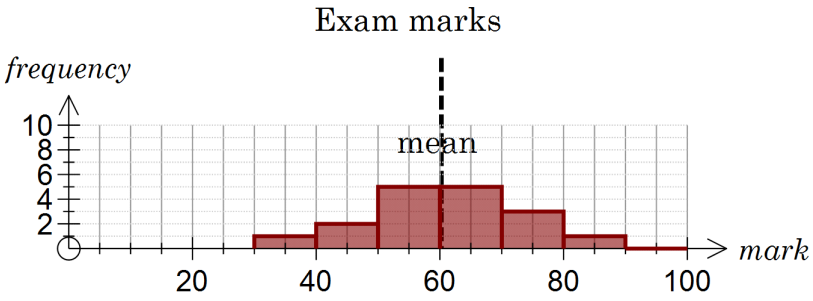
c) The school marks are more spread. The standard deviation is greater as is the inter-quartile range.

2.

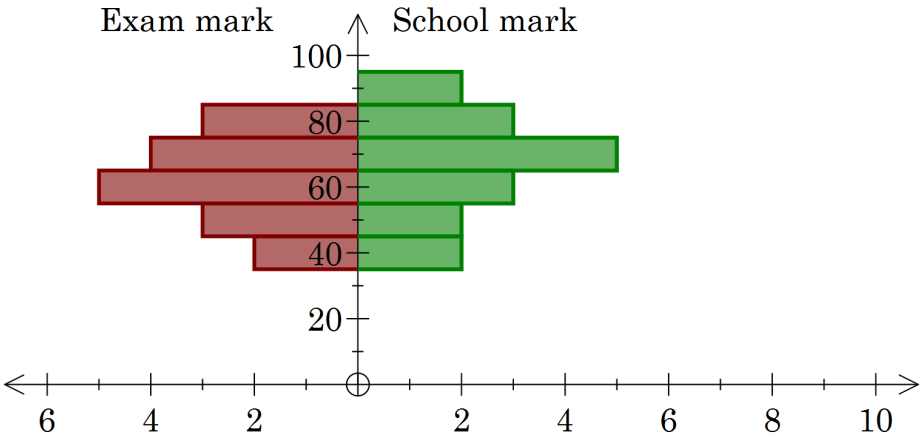
a)



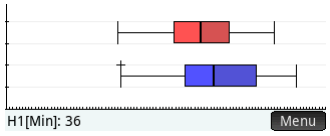




b) This back to back histogram shows the comparison better. School marks are higher and more spread out.



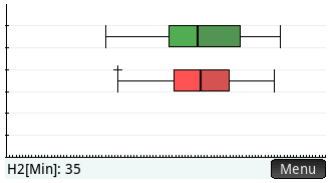
3.

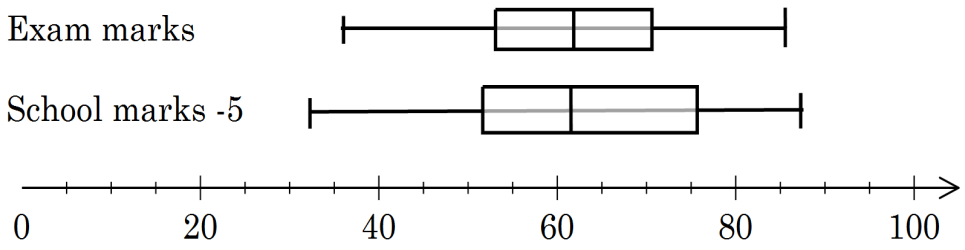
a) 

b) School marks

4.

a) The school marks have been moved down, 5 units to the left on the box plot.





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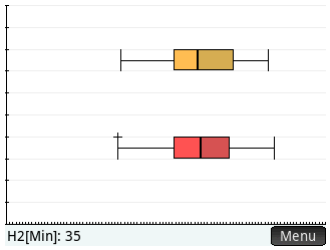
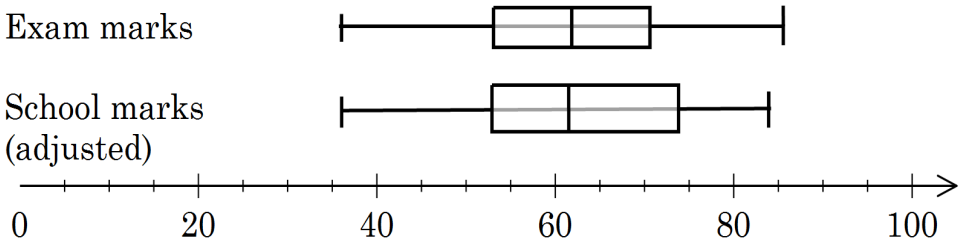
b) 

The exam marks and adjusted school marks have a similar middle and spread/



c) Students B, D, E, F, G, H, L and N did comparatively better in the exam.

	Student																
Mark	A	**B**	C	**D**	**E**	**F**	**G**	**H**	I	J	K	**L**	M	**N**	O	P	Q
School	65	43	78	91	82	65	71	54	61	36	88	69	58	65	79	45	62
Exam	57	**45**	69	**84**	**77**	**61**	**67**	**57**	53	35	75	**65**	52	**62**	71	41	55
Moderated school mark	60	**42**	71	**82**	**74**	**60**	**65**	**51**	57	36	79	**63**	54	**60**	72	44	58

5.

	School mark	Exam mark
Mean	59.1	60.7
Standard deviation	8.57	7.77

a) The students did slightly better in the exam as the mean for the group was higher.

b) The school marks are more spread, higher standard deviation and it can be seen in the table as well with lowest school marks in the 30's but lowest exam marks in the 40's.

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